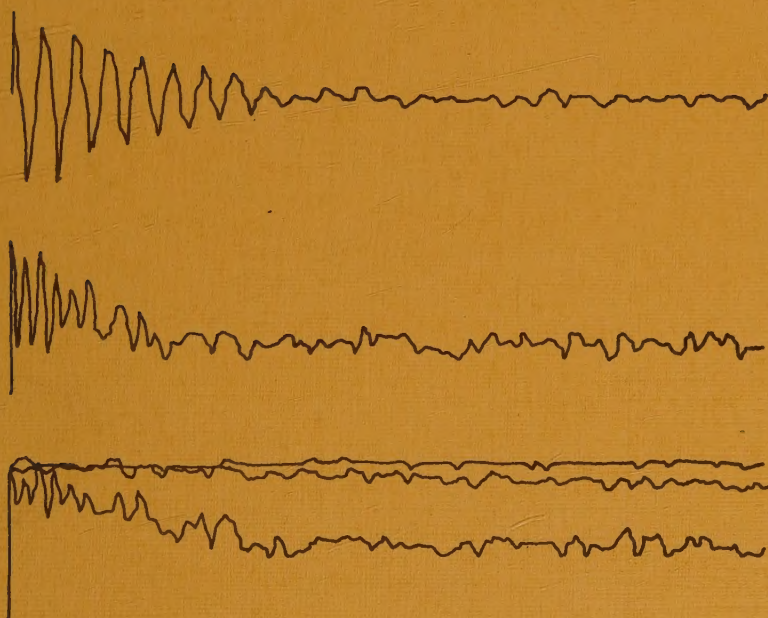


ANALYTICAL MODELS

FOR TRANSMISSION, PROPAGATION AND SPATIAL CORRELATION OF SOUND

SVEN G LINDBLAD

BUILDING ACOUSTICS L T H
FACK 725 S-220 07 LUND



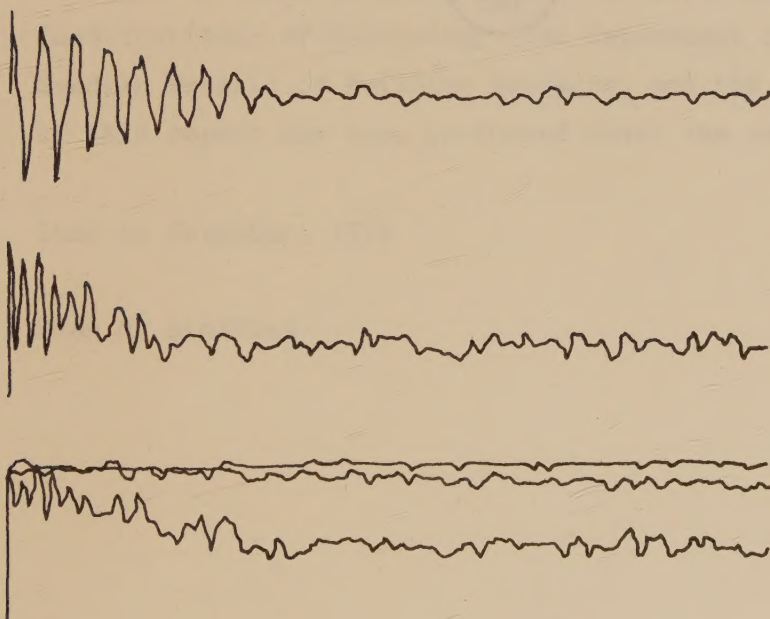
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FOREWORD

This report has been produced during a four months' leave from educational responsibility. I thank Mr Kaj Bodlund for his good management during my research period.

Mr Leif Cederfeldt and Mr Sven-Ingvar Thomasson have shown a great interest in discussions. The questions brought up are depending on the research work of Dr Hans Jonasson, Mr Kaj Bodlund and Dr Anders Nilsson.

Mr Jens Persson and Mr Robert Månsson have performed measurements and Mr Bengt Bengtsson has made a final series of experiments. Mr Carl-Mikael Söderberg has made all the drawings.

I thank all named persons for their support, and especially Miss Monika Johnsson for her never failing interest in interpreting my notes and typing the manuscript.

The work has been produced within the department of Building Acoustics, Lund Institute of Technology. The department receive grants from the Swedish Council of Building Research, and the work forming the basis of this report has been performed under the contracts C 689 and C 865.

Lund in November, 1974

Sven G. Lindblad

SUMMARY

In this work some essential problems in three different acoustic research areas are dealt with.

In the field of transmission the manipulation of narrow spaces and connected waves leads to multiple leaf calculations. A simple edge-scattering power energy model (SPE) is also developed for the reverberant transmission of a partition. Power energy (PE) and mean square coupling (MSC) are compared.

The propagation from a point source is treated by a simple radiation model clearly showing the surface wave as a resonance. The calculations lead to comparisons with other transform methods, which fail to show the surface wave.

The mean square correlation of room fields is discussed and compared with the pressure correlation of travelling waves. A standing wave model results in a new space covariance function for the mean square pressure. This is used as an estimate for the error of the space averaged mean square value.

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THE COVER

The curves show measured auto-correlation of filtered noise, 120-130 Hz, direct, squared and mean square with integration time 1 ms, 0.1 s and 0.5 s. The total τ -axis is 0.2 s. The upper curve of mean square (0.5 s) shows clearly the data reduction in the mean square (Part 4) and the difference from the signal squared (Part 6).

NUMBERING OF EQUATIONS, PARAGRAPHS AND PARTS

The equations are furnished with current numbers within each paragraph. Reference to an equation within the paragraph is made by the current number only, e.g. eq.(12). An equation (e.g. 14) in a preceding paragraph (e.g. par. 5.4) is referred to as eq.(5.4.14).

The paragraphs have current numbers in each part, e.g. par. 5.4 is the fourth paragraph of Part 5. The number of the paragraph is repeated on each page to combine with the equation numbers.

SYMBOLS

Capitals

A	m^2	absorption area
B	Nm	bending stiffness of plate per unit width
B	s^{-1}	bandwidth
C (τ , Δr)		covariance function
C _{mn}		coupling between room - wall
D		mean square
E []		expected value
F	N	force
G(Ω)		transfer function in the PE-model
H _{Ω} (ω)		transfer function
IL	dB	insertion loss
I		"magnitude" intensity
$\vec{J}$		intensity
K	N/m	compliance constant
L	dB	level
M _n		normalizing integral in correlation for transform series
M		type of mode
M		number of measuring points
N		number of modes
O		observation point
(PE)		power-energy (model)
P _n		Fourier term or eigenmode (often pressure)
Q		source field
R(τ , Δr)		correlation function
R	Pa s/m	resistance (velocity)
R	Pa s/m ³	resistance (volume velocity)
S	m^2	area
(SPE)		scatter power energy (model)
T	s	measuring time
T ₆₀	s	reverberation time
TL	dB	transmission loss
U _n		eigenmode (often vibration)
V	m^3	volume of room
W(ω)	m^3	volume velocity transform

Y		ratio of reverberant vibration compared to masslaw vibration
Z	Pa s/m	impedance (velocity)
Z, Z _A	Pa s/m ³	impedance (volume velocity)

Small letters

a,b,d	m	dimensions of a room
c	m/s	velocity of sound
c _B	m/s	bending wave velocity
d	m	air space width
e		exponential base
f	s ⁻¹	frequency
f _c	s ¹	critical frequency
g	m ⁻¹	wave constant
h(t)		impulse response
j		imaginary unit
k	m ⁻¹	wavenumber or transform variable
l	m	length
m,n		often numbering index
p	Pa	pure tone pressure
p(ω)	Pa s	Fourier time transform of pressure
p _n	Pa s	expansion constant in series
q(t)	Pa	sound pressure time function
r	m	distance
$\vec{r}$	m	coordinate in the space
s		radiation factor
t	s	time
u	m/s	pure tone velocity
u(ω)	m	Fourier transform of vibration velocity
v(t)	m/s	vibration velocity time function
w	m	coupling width (strip)
w(t)	m ³ /s	volume velocity

Greek capitals

Γ		ratio of reverberant transmission compared to masslaw transmission
Δ _r		relative mode distance

Θ		expected value of pressure squared in a point
Π	W	power
Π	W/m^2	power per unit area
Σ		summation
Ψ_n		eigenmode, often velocity potential
Ω	s^{-1}	modulation frequency in the power-energy model

Small Greek letters

α		absorption factor
β		relative error of a measured mean value
γ		normalized correlation
δ		normalized covariance
ϵ		relative error
η		loss factor
κ	m^{-1}	transform wavenumber
κ	m^{-1}	Wavenumber of a bending wave
λ	m	wavelength
μ		normalized impedance
μ		expected value
ν		normalized admittance
ξ		often κ/k in transforms
ρc		wave impedance of air
ρ	kg/m^3	density of air
σ^2		variance
τ		transmission factor
τ_e	s	time constant
τ_Ω	s	time constant in PE-analogy
ψ	m	velocity potential
ψ_n	m	expansion factors of the velocity potential
ω	s^{-1}	angular frequency

GENERAL SURVEY

The intention of this paper is to deal with some frequently discussed problems in applied acoustics and the models for simplified solutions.

We shall study some cases, where simplified solutions are often used. My opinion is that improvements and corrections of the customary simple models are of interest.

Some questions have arisen from discussions in papers and colloquia, about the validity of different methods. This is especially the case for the discussions in Part 1. The main interest is devoted to the problem of eigenmodes and vibrating (inhomogeneous) boundaries. Some points of interest in noise transfer and point source power are also discussed, as they will be of importance further on.

Part 2 deals with the wave transforms useful for radiation and also for some propagation problems. I believe that many readers on the postgraduate level are more accustomed to time than space transforms. I shall also introduce some network manipulations I have found to be useful in many types of noise calculations.

In practice it is often necessary to find simple modifications of well-known wave equations. Such modifications and also some connected one-dimensional waves are dealt with in Part 3. The wave in the double leaf with a narrow air space is interesting, as this is an element in multiple leaf partitions. Calculations on a three-glass window with narrow spaces (< 1 mm) are described.

The response of a room to different sources is discussed in Part 4. This leads to comparisons of mode coupling and power energy models. The transmission due to a reverberant field of a partition below and above the critical frequency is discussed. It is shown that power energy methods and the use of infinite half-plates and quarter-spaces leads to a simple and easily understood transmission model. This is claimed to be especially suitable for education purposes.

The use of simple networks combined with the usual radiation transforms in my opinion gives a new picture of the propagation from a

point source on a resilient surface. This is shown in Part 5 and leads to an increased understanding of the conditions for a ducted surface wave. This part is also devoted to comparisons between different transforms and endeavors to make it clear, why some of these transforms fail to show the surface wave.

The sound field in a room driven by filtered white noise has space correlation properties of interest to transmission and power measurements. In Part 6 I claim that it is of great importance to distinguish more clearly than is often done between pressure and mean square correlation. The main result is a new theoretical space covariance function for the mean square pressure. This function seems to be in better agreement with measurements than earlier theories.

PART 1

INTRODUCTORY REMARKS ON TIME FUNCTIONS, WAVES AND INHOMO-
GENEOUS (DRIVING) BOUNDARIES

1.1. Introduction

In this introductory part we shall deal with some well-known methods. The conditions for their application have been discussed in connection with some published papers. I therefore take the opportunity of discussing some points on spectral power transfer, inhomogeneous boundary conditions and eigenmodes used as Fourier terms.

The part begins with some motivations of time function transforms and mean squares and the transfer of the power density spectrum in practice.

After that a paragraph on waves follows, just to show how the solutions are written in this paper.

In order to introduce the different impedance concepts the point source power is shown, together with some networks.

1.2. Some remarks on the acoustical power transfer and the use of symbols

The time histories of pressure and velocity are denoted $q(t)$ and $v(t)$ respectively. It is very important to distinguish q^2 and v^2 , the squared time histories, from the corresponding mean square estimates $\overline{q^2}$ and $\overline{v^2}$. In practice $\overline{q^2}$ and $\overline{v^2}$ are obtained from time integration. The rich signal information is of course reduced by the integration. We denote the true RMS-value by μ_{q2} and its estimate $\hat{\mu}_{q2}$:

$$\hat{\mu}_{q2} = \overline{q^2}(t) = \frac{1}{T} \int_{t-T}^t q^2(t_1) dt_1 \quad (1)$$

The correlation function R is quite different for $\hat{\mu}_{q2}$ and q , as part of the "phase information" is eliminated. (E denotes the statistically expected value.)

$$\begin{aligned} R_{\overline{q^2}}(\tau) &= E[\overline{q^2}(t) \cdot \overline{q^2}(t + \tau)] \\ R_q(\tau) &= E[q(t) \cdot q(t + \tau)] \end{aligned} \quad (2)$$

I believe that q^2 and $\overline{q^2}$ are not always distinguished properly (cp. [1] and [2]).

In linear systems the solution is often built up by transfer functions $H(\omega)$ for the whole system or for the different stages. $H(\omega)$ is the Fourier transform of the impulse response $h(t)$:

$$H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt \quad (3)$$

The symbolic time function $e^{j\omega t}$ is in this report used with the exponent $+j\omega t$. A mass reactance is positive and a compliance reactance is negative in the notation used.

A very simple transfer function is the velocity of a plate driven by a pressure at the surface. p and u denote the Fourier transforms of q and v divided by $\sqrt{2}$:

$$\begin{aligned}
 p &= \frac{1}{\sqrt{2}} \int \exp(-j\omega t) q(t) dt; & u &= \frac{1}{\sqrt{2}} \int \exp(-j\omega t) v(t) dt \\
 u &= \frac{p}{j\omega M}
 \end{aligned} \quad (4)$$

The solution of applied noise transmission problems is often reduced to the mean square response or power response of the total linear system. The input is given by the power density spectrum $S_1(\omega)$ and the output is correspondingly $S_2(\omega)$. $|H(\omega)|^2$ connects S_1 and S_2 :

$$S_2(\omega) = |H(\omega)|^2 \cdot S_1(\omega) \quad (5)$$

(S can be a total or filtered spectrum.)

Using the simple example we get $S_p(\omega) = |p(\omega)|^2$ and $|u(\omega)|^2 = S_u(\omega)$:

$$S_u(\omega) = \left(\frac{1}{\omega M}\right)^2 S_p(\omega) \quad (6)$$

$S(\omega)$ can also be regarded as the transform of the correlation function

$$S(\omega) = \int_{-\infty}^{\infty} R(\tau) \cdot e^{-j\omega\tau} d\tau \quad (7)$$

The connections above are very important, as $H(\omega)$ can be regarded as the response of the scanning pure tone. Hence the response curve can be used also for noise. This fact has sometimes been doubted in practical noise work, e.g. for traffic noise transmission above a grass ground. The case I refer to was a discussion about the presentation of the attenuation curves in [7] at a traffic noise symposium at DTH, Copenhagen, in 1971.

The power per area unit with $\bar{I} = \Pi/S$ of a wave can be estimated by

$$\frac{\bar{\Pi}}{S} = \bar{I} = \frac{1}{T} \int_0^T q(t) \cdot v(t) dt \quad (8)$$

If the transform values are known the intensity $\bar{I}$ for the band (ω_1, ω_2) is obtained from the following (* denotes complex conjugate):

$$\bar{I} = \frac{1}{2\pi} \int_{\omega_1}^{\omega_2} \operatorname{Re}(p \cdot u^*) d\omega \quad (9)$$

If the wave impedance is real ($= \rho c$) we get

$$\bar{I} = \frac{1}{2\pi} \int_{\omega_1}^{\omega_2} |p|^2 / \rho c \cdot d\omega = \frac{1}{2\pi} \int_{\omega_1}^{\omega_2} S_p(\omega) / \rho c \cdot d\omega \quad (10)$$

By using $\sqrt{2}$ in the transforms p and u the factor $1/2$ is eliminated in the entire report. Otherwise the factor is present in MS-calculations.

In mean square coupling (MSC) and power energy models (PE) the total transfer function for the system is not used. Instead powers or mean squares according to eq.(9) are performed **step by step from simpler** transfer functions of the different stages.

1.3. The applied fundamental wave equation

We shall use the simple linear wave equation for air in the following forms ($\Delta = \nabla^2$):

$$\Delta p + \left(\frac{\rho}{\gamma p_0} \right) \omega^2 p = 0 \quad (1)$$

$$\left(\frac{\rho}{\gamma p_0} \right) = \frac{1}{c^2} \quad (2)$$

The one-dimensional x-dependence becomes

$$\exp(\pm jk, x) \quad (3)$$

$$k_1^2 p - \left(\frac{\omega}{c} \right)^2 p = 0 \quad (4)$$

$$\left(\frac{\omega}{c} \right) = k \quad (5)$$

$$k_1^2 p - k^2 p = 0 \quad (6)$$

Solution:

$$k_1 = \pm k \quad (7)$$

(6) is written in this form to point out that k_1 stands for space and k for time derivation. This should be observed in the later parts of this work.

For bending waves we use the simple equation

$$B\Delta(\Delta u) - M\omega^2 u = 0 \quad (8)$$

With sinusoidal x-dependence in one dimension we obtain

$$\exp(-jk, x) \quad (9)$$

$$Bk_1^4 u - M\omega^2 u = 0 \quad (10)$$

With

$$\omega^2 \frac{M}{B} = \kappa^4 \quad (11)$$

we get

$$\kappa_1^4 u - \kappa^4 u = 0 \quad (12)$$

$$\kappa_1 = \pm \kappa \quad (13)$$

$$\kappa_1 = \pm j\kappa \quad (14)$$

In the following we shall without comment change between the "frequency" notations f , ω , k , κ to get the shortest and most convenient expressions.

The symbols of this paragraph are used throughout the report.

By using Snell's law and setting $k_x = \kappa$ we get for radiation (x and y in the surface, z perpendicular):

$$\kappa^2 + k_z^2 = k^2 \quad (15)$$

The boundary condition is

$$\frac{1}{j\omega\rho} \frac{\partial p}{\partial z} = -u_z \quad (16)$$

or

$$\frac{1}{j\omega\rho} \cdot jk_z = u_z \quad (17)$$

and with

$$k = \frac{\omega}{c} ; \quad \omega = kc$$

we get

$$\frac{k_z}{k} \cdot \frac{p}{\rho c} = u \quad (18)$$

From (15) and (18) we obtain

$$Z_{\text{load}} = \frac{p}{u} = \rho c k / k_z = \frac{\rho c}{\sqrt{1 - (\kappa/k)^2}} \quad (19)$$

ρc is called the wave impedance.

As $\kappa = k \sin \phi$ for $\kappa < k$ (ϕ is the angle to the surface normal) we get

$$Z_{\text{load}} = \rho c / \cos \phi \quad (20)$$

Sometimes the velocity potential ψ is used:

$$u = -\nabla \psi \quad (21)$$

$$p = j\omega \psi \quad (22)$$

With ψ the boundary conditions become simpler, e.g. instead of (16):

$$\frac{\partial \psi}{\partial z} = -u_z \quad (23)$$

For the waves we can summarize:

$$\text{Wavenumbers} \quad \left\{ \begin{array}{l} k(\omega) \\ \kappa(\omega) \end{array} \right. \quad (24)$$

$$\quad \quad \quad (25)$$

$$\text{Wave velocities} \quad \left\{ \begin{array}{l} c(\omega) = \omega/k \\ c_B(\omega) = \omega/\kappa = \sqrt{\omega \sqrt{B/M}} \end{array} \right. \quad (26)$$

$$\quad \quad \quad (27)$$

$$\text{Wavelengths} \quad \left\{ \begin{array}{l} \lambda(\omega) = c/f = 2\pi/k \\ \lambda_B(\omega) = c_B/f = \frac{2\pi}{\kappa} \end{array} \right. \quad (28)$$

$$\quad \quad \quad (29)$$

In comments and equations we shall substitute according to the relations above, without further comments.

$k \rightarrow \kappa$ in (19) means that $Z_{\text{load}} \rightarrow \infty$. This is the critical coincidence, where

$$k_c = \kappa_c \quad (30)$$

$$c_c = c_{Bc} \quad (31)$$

$$\lambda_c = \lambda_{Bc} \quad (32)$$

at

$$\omega = \omega_c ; \quad f = f_c \quad (33)$$

k and κ are often given in the following forms:

$$k = k_c (f/f_c) = k_c (\omega/\omega_c) \quad (34)$$

$$\kappa = \kappa_c \sqrt{f/f_c} = \kappa_c \sqrt{\omega/\omega_c} \quad (35)$$

and

$$\kappa = k \sqrt{f_c/f} \quad (36)$$

k and κ are not always connected with real waves as they are used in space transforms of source fields.

1.4. The mean square error, correlation and Fourier terms

In acoustical problems the solution is often obtained by finding a suitable Fourier series in the space domain.

We state that the sum of terms shall give a minimum mean square error. The series $g_f(x)$ replaces the function $g(x)$:

$$g_f(x) = \sum g_n G_n(x) \quad (1)$$

G_n are functions and g_n amplitudes (multipliers) of the terms. The mean square error on the interval $(0, x)$ is

$$\begin{aligned} D &= \int_0^{x_1} (\sum g_n G_n(x) - g(x))^2 dx = \\ &= \int_0^{x_1} (\sum g_n G_n(x))^2 - 2g(x) \sum g_n G_n(x) + g^2(x) dx \end{aligned} \quad (2)$$

Our free multipliers are g_n (Lagrange), so we minimize by derivating on g_n and putting this equal to zero. We derivate on one of the factors g_n and denote the factor g_m :

$$\int_0^{x_1} (\sum g_n G_n(x)) G_m(x) dx - \int_0^{x_1} g(x) \cdot G_m(x) dx = 0 \quad (3)$$

Now our problem is how to handle the sum. If G_n is chosen such as

$$\int_0^{x_1} G_n(x) G_m(x) dx = 0 \quad \text{if } m \neq n \quad (4)$$

our problem is solved. Hence we get

$$g_m = \frac{\int_0^{x_1} g(x) G_m(x) dx}{\int_0^{x_1} G_m^2(x) dx} \quad (5)$$

The uncorrelated functions G_n are called orthogonal on $(0, x_1)$ analogous

with vectors. The interval is important, and we go straight to the simple trigonometric series. On the interval $(0, 1)$ we have

$$G_n = \sin(2\pi n x/1) ; \cos(2\pi n x/1) \quad (6)$$

or

$$G_n = \sin(\pi n x/1) \quad (7)$$

or

$$G_n = \cos(\pi n x/1) \quad (8)$$

If the terms involve complete periods as in (6) sine is orthogonal to cosine, but with half periods this is not the case. Consequently (7) and (8) are also complete series.

In the time domain (6) is frequently used, as the definition range is not $(0, 1)$ but has a given periodical extension. (7) and (8) will give an image + periodical extension. The image reproduces the boundary condition, so (7) and (8) are often used in the space domain.

We can also use the whole period series (6) but use $2l$ as an interval.

If $(0, 1)$ is the definition interval the even and odd extension can give (8) and (7). Odd and even functions only produce sine and cosine terms respectively. This is all clear from the following figures.

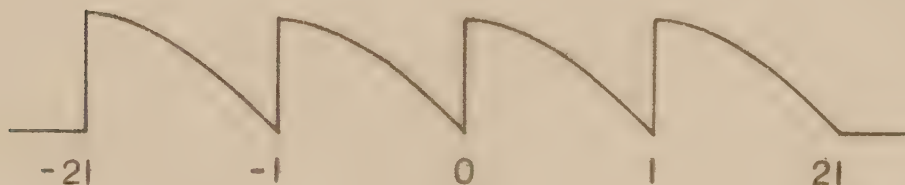


FIGURE 1.4.1.a. On $(0, 1)$ sine + cosine terms.

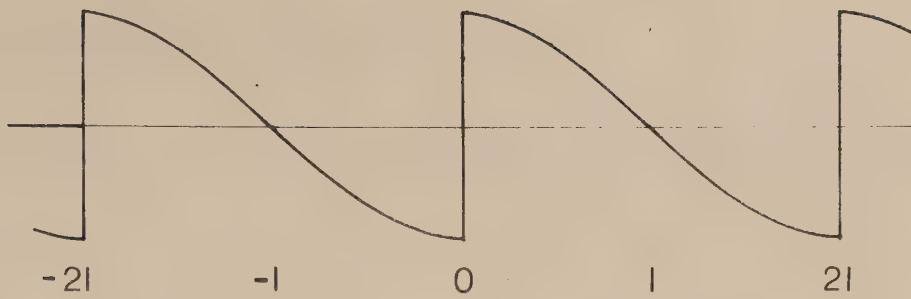


FIGURE 1.4.1.b. On $(0, 2l)$ with odd image, only sine terms.

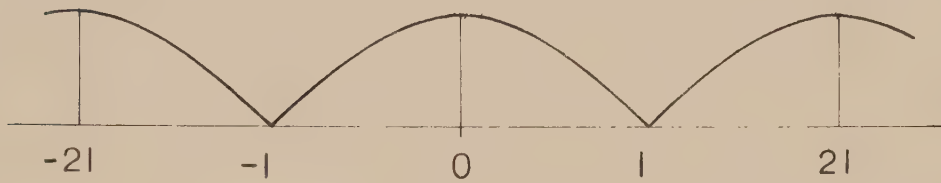


FIGURE 1.4.1.c. On $(0, 2l)$ with even image, only cosine terms.

One reason to show this here is that a function starting with a step gives a symmetrical step together with the odd image. In a step the series, according to Fourier's theory, gives the mean value of x^+ and x^- , which is zero for the odd image sine-series. This should be observed in the following discussion of boundary conditions.

There are also other orthogonal series, which can be used as Fourier series, e.g. with eigenmodes as terms. For simple Cartesian forms and simple boundaries these series can be trigonometric functions.

1.5. The tube with driving piston

In a tube with a hard piston at $x = 0$ and a hard wall at $x = l$ the particle velocity $u(x)$ is solved in the following direct manner:

$$u(0) = u_0 \quad (\text{piston velocity}) \quad (1)$$

$$u(l) = 0 \quad (2)$$

$$\Delta u + k^2 u = 0 \quad (3)$$

$$u = u_{+-} \cdot e^{\pm jkx} \quad (4)$$

As $u = u_+ + u_-$ is zero at $x = l$ we get a standing wave solution $u_s(x)$ (s for standing wave):

$$u_s(x) = u_0 \frac{\sin(k(l-x))}{\sin(kl)} \quad (5)$$

This is a typical case of an inhomogeneous (driving) boundary at $x = 0$. The solution $u_s(x)$ fulfils this condition.

Now we shall transform $u_s(x)$ into a Fourier series $u_f(x)$. Among the possible series we choose the sine-terms, as these terms are the eigenmodes with homogeneous hard boundaries (f for Fourier series, cf. eq. (1.4.5)):

$$u_f(x) = \frac{u_0 \pi}{l^2} \sum_{n=-\infty}^{\infty} \frac{-n}{k^2 - (n\pi/l)^2} \sin(n\pi x/l) \quad (6)$$

u_s and u_f obtain poles at $k = (n\pi/l)$ but we introduce losses as a loss factor η and replace k by $\bar{k}$ to get finite maxima resonances:

$$k^2 \rightarrow \bar{k}^2 = k^2(1 - j\eta) \approx (k(1 - j\eta/2))^2 \quad (7)$$

η can also be looked upon as a description of the relative energy loss of vibrating systems:

$$\eta = \frac{\text{energy loss/radian}}{\text{energy}} \quad (8)$$

We also have a complex wave constant g that can be useful as an alternate form of the complex wave constant:

$$g = j\bar{k} = k\eta/2 + jk \quad (9)$$

Near a resonance we expand k :

$$k = (n\pi/l) + \Delta k \rightarrow k^2 \approx (n\pi/l)^2 (1 + 2\Delta k/k) \quad (10)$$

At a resonance we obtain a peak function:

$$\begin{aligned} \frac{u'_s}{u_o} &= \frac{\sin h(g(1-x))}{\sin h(gl)} = \frac{\sin h(g(1-x))}{\sin h(kl\eta/2) \cos(kl) + j \cos h(kl\eta/2) \sin(kl)} \\ &\approx \frac{\sin h(g(1-x))}{kl\eta/2 + j\Delta kl} = \frac{\sin h(g(1-x))}{\eta/2 + j\Delta k/k} \cdot \frac{1}{kl} \end{aligned} \quad (11)$$

The last approximative forms are true for $\Delta k \ll \pi/l$ only. From u'_f we get in the same range one dominating term if x is far from 0:

$$u'_f \approx \frac{u_o/(kl)}{\Delta k/k + j\eta/2} \sin(n\pi x/l) \quad (12)$$

(12) is a usual method to get u at resonance, but we see that u'_f cannot fulfil the boundary condition as $u'_f = 0$ at $x = 0+$. As a matter of fact no term dominates alone.

To complete the picture we study u_s and u_f for $kl = \pi/2$. We obtain:

$$u_s(x) = u_o \frac{\sin(\pi(1-x)/(2l))}{1} \quad (13)$$

$$u_f(x) = \frac{u_o}{\pi} \sum_{n=-\infty}^{\infty} \frac{+n}{n^2 - 1/4} \sin(n\pi x/l) \quad (14)$$

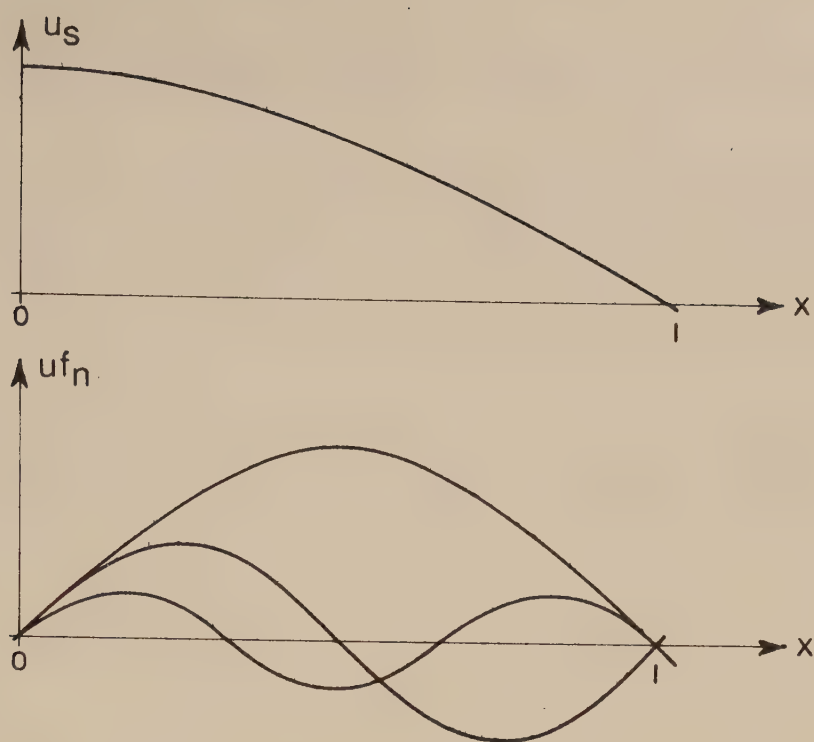


FIGURE 1.5.1. Standing wave and Fourier terms.

1.6. Inhomogeneous boundary conditions from homogeneous boundaries

The usual way to solve a differential equation with homogeneous boundary conditions is to transform the differential equation into a Fourier series, which for every term fulfils the boundary condition, the eigenmodes. As a result of this choice also the series will fulfil the boundary condition for any combination of terms.

The inhomogeneous differential equation has a source distribution Q and usually the velocity potential ψ is used ($-u = \partial\psi/\partial x$):

$$\frac{\partial^2 \psi(x)}{\partial x^2} + k^2 \psi(x) = Q(x) \quad (1)$$

The character of Q is a distributed increment to the divergence (sources) $\partial u/\partial x$. The eigenmodes Ψ_n are solutions of (1) with $Q = 0$:

$$\frac{\partial^2 \Psi_n(x)}{\partial x^2} = -k_n^2 \Psi_n(x) \quad (2)$$

$$\Sigma \Psi_n (-k_n^2 + k^2) \Psi_n(x) = Q(x) \quad (3)$$

Multiplying both sides with Ψ_m , integrating and using the orthogonality of Ψ_n we obtain

$$\psi_m = \frac{\int_0^1 \Psi_m(x) \cdot Q(x) dx}{(-k_n^2 + k^2) \int_0^1 \Psi_m^2(x) dx} \quad (4)$$

With $\int |\Psi_n|^2 dx = M_{\Psi n}$ we transform Q on Ψ_n :

$$q_{\psi n} = \frac{\int_0^1 Q(x) \Psi_n(x) dx}{M_{\psi n}} \quad (5)$$

This is a usual Fourier series. We can now write the solution of ψ with

the source Fourier terms:

$$\psi(x) = \sum \psi_n \psi_n(x) \quad (6)$$

$$\psi_n = \frac{q_{\psi n}}{-k_n^2 + k^2} \quad (7)$$

In a tube $\psi_n = \cos k_n x$ and $u = -\partial\psi/\partial x$, which gives

$$u(x) = \frac{-\partial\psi}{\partial x} = \frac{-\partial}{\partial x} \sum \frac{\int_0^1 Q(x) \cos(n\pi x/l) dx}{(-k_n^2 + k^2) M_{\psi n}} \cdot \cos(n\pi x/l) \quad (8)$$

If $Q(x)$ is a point source with the one-dimensional volume velocity $W = u_0$, we use the Dirac $\delta(x - x_1)$. Now we can treat the inhomogeneous boundary discussed earlier by a source at $x = 0$ and homogeneous boundary. With $Q(x) = 2 \delta(x)$. We get the following (2δ is due to the image field):

$$u = -\frac{\partial}{\partial x} \sum \frac{W \cos(n\pi x/l)^2}{(-k_n^2 + k^2) M_{\psi n}} \quad (9)$$

If $x > 0$ (9) can give the solution of u . This is one way to obtain a series solution (u_f). This method was used in [5] and then summed up to give a standing wave solution. It could have been more direct and avoided discussions of fulfilling the boundary condition, to get u_s directly in that case, as shown in par. 1.5.

The transient solution u_t has homogeneous boundaries and a homogeneous differential equation. With b'_n depending on the conditions at $t = 0$ we obtain a series of eigenmodes for $t > 0$. This is the only series in this case:

$$u_t(t) = \sum_{-\infty}^{\infty} b'_n \cdot e^{-\frac{n}{2} \omega_n t} \cdot e^{j\omega_n t} \sin(k_n x) \quad (10)$$

The terms have different oscillation frequencies ω_n , identical with the resonance frequencies.

In [3] a solution of standing wave form + transient form ($u_s + u_t$) was given. The authors also referred to [4] claiming that the solution for steady state given in [4] as a Fourier series (u_f) was a transient solution u_t . It seems as if this was caused by the solution of [4] given in u_f -form and then confused with the authors' u_t -part of their ($u_s + u_t$)-solution.

The discussions about the different forms of solutions, especially concerning the fulfilling of inhomogeneous boundary conditions and the different u_s -, u_f - and u_t -solutions, are important to the comprehension of this problem. Often the differences have minor importance in practice, as we look for mean square (MS) results and introduce losses to handle the poles.

Often the mean square (MS) result of the resonances dominates the MS-solution. Thus problems with convergence between resonances will not occur in MS-calculations.

1.7. The plate vibrations with homogeneous boundaries

With a pressure distribution $p(x, y)$ we get for a plate:

$$(\Delta \cdot \Delta - \kappa^4) u(x, y) = -j\omega p(x, y)/B \quad (1)$$

The eigenmodes of the plate vibration U_n also fulfil

$$\Delta(\Delta U_n) = \kappa_n^4 U_n(x, y) \quad (2)$$

$$u(x, y) = \sum u_n U_n(x, y) \quad (3)$$

We insert (3) in (1), multiply by one term and integrate. From the orthogonality of U_n -terms we get

$$u_n = \frac{j\omega}{B} \frac{\iint p(x, y) U_n(x, y) dx dy}{\iint U_n^2(x, y) dx dy (-\kappa_n^4 + \kappa^4)} \quad (4)$$

For the norm integral we use M_{un} and introduce the Fourier series of p on U_n , p_{un} :

$$p_{un} = \frac{\iint p(x, y) U_n(x, y) dx dy}{M_{un}} \quad (5)$$

The vibration can now be expressed in the Fourier term p_{un} as

$$u_n = \frac{p_{un}}{-\kappa_n^4 + \kappa^4} \frac{j\omega}{B} \quad (6)$$

The " u_n "-series is a Fourier series of the driving pressure in terms of the eigenmodes of the driven system.



$$\frac{\partial^2 (px)}{\partial x^2} + k^2 (px) = 0 \quad (5)$$

$$p = p_0(x_0/x) \exp(-jk(x - x_0)) \quad (6)$$

The driving source can be a pulsating sphere with the radius r . A vibration velocity u_0 gives a volume velocity W :

$$W = 4\pi r^2 u_0 \quad (7)$$

From (1) and (6) we obtain:

$$p(1/x + jk) = jk\rho c u \quad \text{for } x \geq r \quad (8)$$

At the surface we get the sound pressure p_0 :

$$p_0 = \frac{jk\rho c u_0}{1/r + jk} = \frac{Wk\rho c}{4\pi} \cdot \frac{k + j/r}{1 + (kr)^2} \quad (9)$$

The pressure p for $x \geq r$ will be:

$$p = p_0 \cdot \frac{r}{x} \exp(-jk(x - r)) \quad (10)$$

The pressure from a point source is obtained for $r \rightarrow 0$:

$$p = j \frac{Wk\rho c}{4\pi x} \exp(-jkx) = \frac{Wk\rho c}{4\pi x} (\sin(kx) + j \cos(kx)) \quad (11)$$

The radiated power is

$$\Pi = \operatorname{Re}(W^* p(0)) = |W|^2 k^2 \rho c / (4\pi) \quad (12)$$

The simple point source has been brought up here as we shall use point source transforms later in radiation and propagation problems.

1.9. Equivalent networks

Networks that are equivalent to the equations are of great help when doing restricted but important alterations. The network can be regarded as an equivalent electrical circuit but this is not necessary.

We get the pressure "drop" $\Delta p = uZ$ for a wall, where Z can be $j\omega M$ according to the masslaw. We can directly use u and p in the network as flow and pressure in the following way:

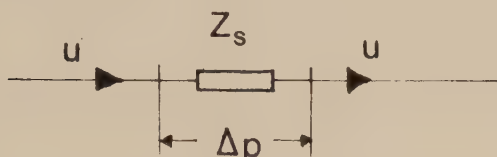


FIGURE 1.9.1. The pressure drop.

A pressure source can have acoustical one-port impedance Z_g as in the following figure.



FIGURE 1.9.2. The pressure source velocity pressure one-port.

A velocity source gives the one-port below.

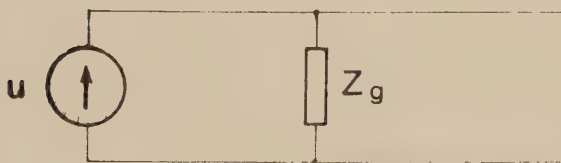


FIGURE 1.9.3. The velocity source one-port.

The radiation from a plane surface with resilient covering Z_g is given by the network in FIG 1.9.4, where u_0 is the hard wall velocity.



FIGURE 1.9.4. The velocity source shunted by a resilient layer.

The radiated power Π is $u_2^2 R_s$ per area unit ($\Pi/S = I$):

$$I = |u_2|^2 R_s = |u_0|^2 \left| \frac{Z_g}{Z_g + R_s} \right|^2 R_s \quad (1)$$

Networks can also be given for force and velocity (mechanical) and for volume velocity and pressure. The last type will be used in this work, e.g. for a pulsating spherical source.

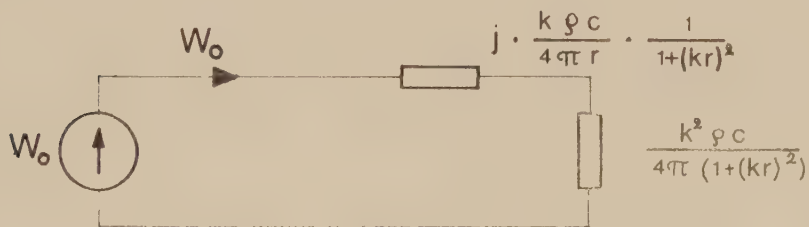


FIGURE 1.9.5. Acoustical volume flow network. Impedance: p/W .

1.10. Concluding remarks to Part 1

My hope is that Part 1 will give the postgraduate level reader an increased perception and comprehension of waves and Fourier series by means of the simple cases chosen. The choice of subjects reflects my own education. I have shown the results here the way I should have preferred to have them demonstrated to myself. Fourier calculations should not be some values of manipulation, but really be based on the physical assumptions and conditions.

PART 2

RADIATED POWER AND TRANSMISSION NETWORKS

2.1. Introduction

Some special effects of infinite and limited areas on radiation will be discussed in this part. Some network methods and the wave two-port are also introduced. Practical points on transmission factors and symmetrical constructions are dealt with. The idea is to obtain a simplified picture of some more or less well-known solutions.

Also this part has an introductory character, but it includes discussions about the different methods, that should be of interest to the advanced reader.

Results concerning grazing radiation will be used further on.

The simple point source radiation is dealt with in this part, as a radiation model will be introduced in Part 5.

2.2. Fourier transform to "line waves"

The benefit of a time frequency transform is often that the response of the pure tone can be used for the transformed variable as well. Similarly the Fourier wavenumber transform presents the possibility of using pure line wave response for any form of vibration of the plane source. We use κ as a transform variable on an area with the vibration $u(\omega, x, y)$ and get the corresponding transform u_{κ} :

$$u_{\kappa}(\kappa_x, \kappa_y) = \frac{1}{2\pi} \cdot \iint_{-\infty}^{\infty} u(x, y) \exp(j(\kappa_x x + \kappa_y y)) dy dx \quad (1)$$

$u_{\kappa}(\kappa_x, \kappa_y)$ can be interpreted as a wave with the following direction of propagation, a line wave:

$$\text{arc tg}(\kappa_y/\kappa_x) \quad (2)$$

The inverse transform is

$$u(x, y) = \frac{1}{2\pi} \iint_{-\infty}^{\infty} u_{\kappa}(\kappa_x, \kappa_y) \exp(-j(\kappa_x x + \kappa_y y)) d\kappa_x d\kappa_y \quad (3)$$

The line wave produces a sound field p_{κ} that must fulfil the wave equation p_{κ} :

$$(k_x^2 + k_y^2 + k_z^2 - k^2) p_{\kappa} = 0 \quad (4)$$

The boundary condition along the surface is

$$k_x = \kappa_x, \quad k_y = \kappa_y \quad (5)$$

$$k_z = \sqrt{k^2 - \kappa^2} = k\sqrt{1 - \xi^2}$$

$$\left. \begin{aligned} \kappa_x^2 + \kappa_y^2 &= \kappa^2 = \xi^2 k^2 \end{aligned} \right\} \quad (6)$$

$$\xi = \kappa/k$$

The boundary condition in normal direction is

$$u_{\kappa} = -\frac{\partial p_{\kappa}}{\partial z} \cdot \frac{1}{j\omega\rho} = \frac{k_z p_{\kappa}}{k\rho c} = \frac{\sqrt{1-\xi^2}}{\rho c} p_{\kappa} \quad (7)$$

Hence the load impedance will be

$$Z = p_{\kappa}/u_{\kappa} = \frac{\rho c}{\sqrt{1-\xi^2}} \quad (8)$$

Z is real for $\kappa < k$ and the radiation resistance can be written

$$R = \rho c / \cos \phi \quad (9)$$

With $\kappa > k$, Z is imaginary. $\text{Im}(Z)$ is a mass reactance, as we must define $\sqrt{1-\xi^2}$ as $-j\sqrt{\xi^2-1}$ for $\xi \geq 1$, to get a not growing radiated wave:

$$Z = j\rho c / \sqrt{\xi^2-1} \quad (10)$$

We get the reactive, short circuited near field

$$p_{\kappa}(z) = ju \frac{\rho c}{\sqrt{\xi^2-1}} \exp(-kz\sqrt{\xi^2-1}) \quad (11)$$

2.3. Point source and unlimited line source on a wall

We have a point source W_3 (m^3/s) and a line source W_2 (m^2/s). Analogous with the white spectrum for a short pulse the point source gives a white κ -spectrum and we obtain

$$u_{3\kappa} = \frac{1}{2\pi} \iint W_3 \cdot \delta(x, y) \exp(j(\kappa_x x + \kappa_y y)) dx dy = W_3/(2\pi) \quad (1)$$

and

$$u_{2\kappa} = \frac{1}{\sqrt{2\pi}} \int W_2 \cdot \delta(x) \exp(j\kappa_x x) dx = W_2/\sqrt{2\pi} \quad (2)$$

We integrate the power of u_{κ} and get the totally radiated power

$$\begin{aligned} \Pi_3(k) &= \frac{\rho c W_3^2}{(2\pi)^2} \int_0^{2\pi} \int_0^k \frac{1}{\sqrt{1 - (\kappa/k)^2}} \kappa d\kappa d\psi = \\ &= W_3^2 \frac{k^2 \rho c W_3^2}{2\pi} \int_0^1 \frac{1}{\sqrt{1 - \xi^2}} \xi d\xi = W_3^2 \frac{k^2 \rho c}{2\pi} \end{aligned} \quad (3)$$

(with $\kappa_x = \kappa \cos \psi$, $\kappa_y = \kappa \sin \psi$ and $d\kappa_x d\kappa_y = \kappa d\kappa d\psi$)

The line source produces the following power per length unit:

$$\begin{aligned} \Pi_2(k) &= \frac{\rho c W_2^2}{2\pi} \int_{-k}^k \frac{1}{\sqrt{1 - (\kappa/k)^2}} d\kappa = \\ &= \frac{k \rho c W_2^2}{2\pi} \int_{-1}^1 \frac{1}{\sqrt{1 - \xi^2}} d\xi = W_2^2 \frac{k \rho c}{2} \end{aligned} \quad (4)$$

Π_3 can easily be derived directly (par. 1.8), but for Π_2 the transform method is suitable.

Let us compare with Rayleigh's radiation formula, which integrates the result from point source elements on the wall. From eq.(1.8.11) we directly get

$$p_{\text{point}} = \frac{jk\rho c W_3}{4\pi r_1} \exp(-jkr_1) \quad (5)$$

The line is an integral of points, that is ($r_1 = \sqrt{r^2 + x^2}$):

$$p(r) = \int_{-\infty}^{\infty} \frac{jk\rho c W_2}{4\pi \sqrt{r^2 + x^2}} \exp(-jk\sqrt{r^2 + x^2}) \cdot dx \quad (6)$$

The difference between point sources and sources in the form of "line waves" (in the transform) is that the former depend on each other and have mutual radiation resistances, while the latter are orthogonal with respect to the radiation resistance.

It should be mentioned that the transformation that gives the pressure of a certain point also can be complicated, but for calculations of radiated power it is very convenient.

[8] is an early reference, in which the transforms are used to obtain the radiation resistance. For plates of different shapes the transform methods were used in [9]. This work has led to the knowledge about edge-radiation.

2.4. The radiation from a strip

The radiation from an unlimited strip with the width l , surrounded by a hard surface, can be calculated as follows (u_0 is a constant velocity):

$$u_{\kappa} = \frac{1}{\sqrt{2\pi}} \int_{-l/2}^{l/2} \exp(j\kappa_x x) \cdot u_0 dx = \frac{u_0 l}{\sqrt{2\pi}} \frac{\sin(\kappa_x l/2)}{\kappa_x l/2} \quad (1)$$

The radiated power is Π per length unit of the strip:

$$\Pi = \frac{k u_0^2 \rho c l^2}{2\pi} \int_{-1}^1 \frac{1}{\sqrt{1 - \xi^2}} \left(\frac{\sin(kl\xi/2)}{kl\xi/2} \right)^2 d\xi \quad (2)$$

This result is similar to the spectrum of an ideal pulse in the time - frequency domain. Instead of constant velocity we shall now introduce a vibration wave:

$$u = u_0 \exp(-j\kappa_0 x) \quad (3)$$

The transform is now strongest at $\kappa = \kappa_0$, as could be expected (cf. a spectrum of a pure tone pulse). With $\kappa_0 = k\xi_0$ we get:

$$\Pi = \frac{(u_0 l)^2 \rho c}{2\pi} \int_{-1}^1 \frac{1}{\sqrt{1 - \xi^2}} \left(\frac{\sin(k(\xi - \xi_0) l/2)}{k(\xi - \xi_0) l/2} \right)^2 d\xi \quad (4)$$

Of special interest is the radiation for $\kappa_0 = k$, grazing sound radiation. This can be the case for limp walls with grazing incident sound or at the critical frequency for bending wave fields. The radiation resistance is $R_s = \Pi/|u|^2$. We obtain the grazing resistance R_g :

$$\frac{R_g}{\rho c} = \frac{lk}{2\pi} \int_{-1}^1 \frac{1}{\sqrt{1 - \xi^2}} \left(\frac{\sin(k(\xi - 1) l/2)}{k(\xi - 1) l/2} \right)^2 d\xi \quad (5)$$

If $kl \gg 2\pi$ there are many oscillations of the sine-function. We there-

fore use the mean value 0.5 during the dominating part of integration. We get a simple value, which is a somewhat too large as also $(\xi - 1)^{-1}$ is omitted:

$$\frac{R_g}{\rho c} \approx \frac{lk}{\pi} \int_0^1 \frac{0.5 d\xi}{\sqrt{1 - \xi^2}} = \frac{lk}{4} = \frac{\pi l}{2\lambda} \quad (6)$$

This radiation resistance will be used in Part 3 and Part 4 as a simple correction to $R_s = \rho c / \cos \phi$ for limited transmitting areas. This is a way to obtain a practical, limited R'_s of the form $\sim (1/R_s + 1/R_g)^{-1}$. The underestimate of $1/R_g$ from (6) is compensated by the simple addition of $1/R_s$ and $1/R_g$.

$$R'_s = \frac{\rho c}{\cos \phi + 4/kl} \quad (7)$$

2.5. The "window" in a plane

For later comparisons we shall use the cosine series for the vibrations of an area ab in a hard plane.

$$u(x, y) = \sum \sum u_m \cdot U_m(x, y) \quad (1)$$

$$m = (m_x, m_y)$$

$$U_m(x, y) = \cos \frac{m_x \pi x}{a} \cos \frac{m_y \pi y}{b} = \cos(\kappa_{mx} x) \cos(\kappa_{my} y) \quad (2)$$

$$u_{m_x m_y} = \frac{\iint U_{m_x m_y}(x, y) \cdot u(x, y) \, dx dy}{\iint U_{m_x m_y}^2 \, dx dy} \quad (3)$$

U_m are $(\cos \cdot \cos)$ -terms and not eigenfunctions, unless the window is a single pane with $\partial u / \partial x = \partial^3 u / \partial x^3 = 0$, guided boundaries. The Fourier transform of a cosine term of the vibration of ab on the infinite plane is

$$\begin{aligned} u'_\kappa(\kappa_x, \kappa_y) &= \frac{1}{2\pi} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \cos(\kappa_{mx} x) \cos(\kappa_{my} y) \exp(j(\kappa_x x + \kappa_y y)) \, dy dx = \\ &= \frac{1}{2\pi} (\text{Int } x)(\text{Int } y) \end{aligned} \quad (4)$$

$$(\text{Int } x) = (-1)^{m_x} \frac{2\kappa_x \sin((\kappa_x - \kappa_{mx}) a/2)}{\kappa_x^2 - \kappa_{mx}^2} \quad (5)$$

$(\text{Int } y)$ has the same form as $(\text{Int } x)$. This expression is also valid for a strip with the width a for a cosine term. A question that has been discussed is if a cosine form (a real form or a Fourier term) radiates any power for $\kappa_m > k$. According to [9] the power is zero, but in [10] the author claims that the radiated power is nonzero. We shall discuss this further on.

The transform component $u'(\kappa)$ can only radiate real power for $\kappa < k$, as

the radiation impedance is

$$Z_s = \frac{\rho c}{\sqrt{1 - (\kappa/k)^2}} \quad (6)$$

Consequently the next question is if $u'_\kappa \neq 0$ for $\kappa < k$, also when $\kappa_{mx}^2 + \kappa_{my}^2 = \kappa_m^2 > k$. These conditions imply that $\kappa_x < \kappa_{mx}$ in eq.(5). This fact prevents full correlation ($\kappa_x = \kappa_{mx}$) but does not give $\text{Int } x = 0$ for all values of κ_x . We therefore conclude that there will be a small but non-zero radiation also for cosine terms below the critical frequency. We should also remember that the case in question concerns radiation to the infinite half-space.

2.6. The room with infinite and finite depth

Now we consider radiation from a surface ab to waves in a room (a, b, ∞) . The following transmission is the same as in wave guides and fulfil the hard boundary conditions.

$$P_n(x, y) \cdot e^{-jk_{nz}z} = \cos(k_{nx}x) \cos(k_{ny}y) \cdot e^{-jk_{nz}z} \quad (1)$$

P_n is the transform series term that directly fulfils the present image extension:

$$p(x, y, z) = \sum \sum p_n P_n(x, y) \cdot e^{-jk_{nz}z} \quad (2)$$

As one dimension is left free we get a continuous solution of eigenvalue:

$$\left. \begin{aligned} k_{nx}^2 + k_{ny}^2 &= k_n^2 \\ k_n^2 + k_{nz}^2 &= k^2 \end{aligned} \right\} \quad (3)$$

$$k_{nz} = \sqrt{k^2 - k_n^2} = k\sqrt{1 - (k_n/k)^2} = k\sqrt{1 - \xi^2} \quad (4)$$

($k_{nx} = n\pi/a$ and $k_{ny} = n\pi/b$.) k_{nz} is imaginary if $k_n > k$. We transform the vibration of the driving wall on $P_n(x, y)$. This is the usual transform of driving field on boundary fulfilling terms of the driven field. This concerns the homogeneous conditions of the flanking walls:

$$u = \sum u_{pn} P_n \quad (5)$$

$$u_{pn} = \frac{\iint u(x, y) P_n(x, y)}{M_{pn}} \quad (6)$$

As P_n is orthogonal each Fourier term u_{pn} is producing its own pressure term p_n from the boundary condition at the driving surface ab :

$$u_{pn} = \frac{p_n}{k\rho c} \cdot k_{nz} \rightarrow p_n = \frac{u_{pn}\rho c}{k_{nz}/k} = \frac{u_{pn}\rho c}{\sqrt{1 - \xi_n^2}} \quad (7)$$

$$Z_s(k, \kappa_{nx}, \kappa_{ny}) = \frac{p_n}{u_{pn}} = \frac{\rho c}{\sqrt{1 - \xi_n^2}} \quad (8)$$

$$p(x, y, z) = \Sigma \Sigma \frac{\rho c u_{pn} P_n(x, y)}{\sqrt{1 - \xi_n^2}} \cdot e^{-k_{nz} z} \quad (9)$$

For $k_n > k$, p_n , z_n and k_{nz} are imaginary. Accordingly we get an attenuated reactive nearfield.

We shall revert to the discussion in par. 2.5. A $(\cos \cdot \cos)$ -term u_m on the wall will only give one u_{pn} -term (with $\kappa_m = \kappa_n$). So if $\kappa_m > k$, $\kappa_n > k$ as well, and the radiated power is zero as Z_s is imaginary for the term.

Some authors seem to have used the result obtained for a room also for an infinite half-space. However, the resulting error is very small. The sine-term radiates considerably for $\kappa_m > k$ both to a half-space and a room. This is due to the fact that the cosine series (n) of one sine-term (m) with $\kappa_m > k$ will also have strong terms with $\kappa_n < k$.

For finite depth l we obtain the solution in standing wave form. As the wavenumber k_z is known and $x = l$ is a hard wall, the standing wave for the pressure is (cf. par. 1.5):

$$p(x, y, z) = \Sigma \Sigma -j P_m(x, y) \frac{u_{pm} \rho c \cdot \cos(k_{mz}(l - z))}{\sqrt{1 - \xi_m^2} \cdot \sin(k_{mz} l)} \quad (10)$$

Also in this case k_{mz} is imaginary for $k_m > k$. A low frequency cosine vibration of the wall will give an attenuated room field in the z -direction only. Z_s is imaginary for all ξ -values. There is no dissipation in the closed room.

For comparison we repeat the eigenmode solution of the wall when driven by the pressure p' of the source room. From Part 1 we obtain:

$$u_n = \frac{\iint U_n(x, y) p'(x, y) dx dy}{M_n \left(-\frac{1}{\kappa_n} + \frac{1}{\kappa} \right)} \quad (11)$$

$$u = \sum u_n U_n \quad (12)$$

To find u_{pn} we must know u . In [6] u_n and u_{pn} are called B_n and ϕ_n . During the deduction u_n and u_{pn} have been confused. A solution is given by u_{pn} in spite of the fact that u is never solved (in u_n) from p .

For some boundaries the eigenmodes U_n have a simple mathematical form. E.g. a simple supported plate has (sin • sin)-terms. The pressure modes of the simple room are (cos • cos • cos). Therefore we often get cos-sin-cos correlation in transmission problems for the steps room 1 - partition - room 2.

In [4] a solution was given for the room pressure from wall vibrations with sliding boundaries. This type of boundary was chosen to get identical cos-eigenmodes of wall and room. However, the solution can be regarded as the response of the room to one term of the Fourier series of the wall on the room modes (u_{pn}). The solution is therefore more general than is claimed by the author.

2.7. The sound wave one-port and the transmission

The sound generator for the primary field with the intensity I^+ can be chosen as a matched generator with $R_y = \rho c$. The secondary field is also terminated for matching. The usual masslaw transmission leads to a network with two lines, according to FIG 2.7.1.

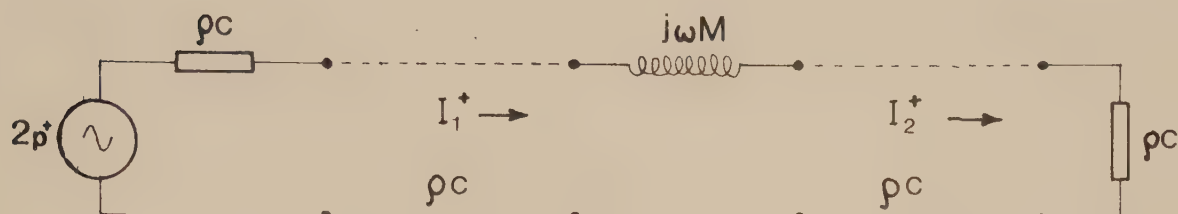
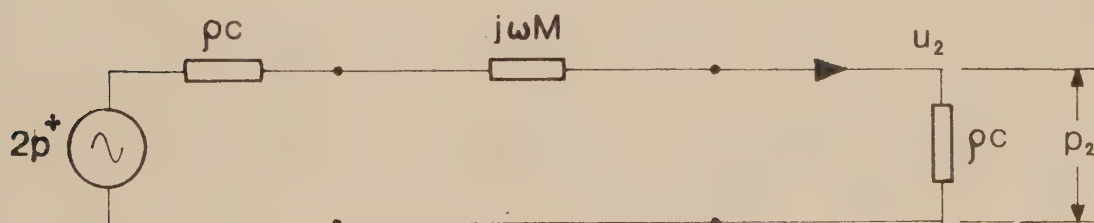


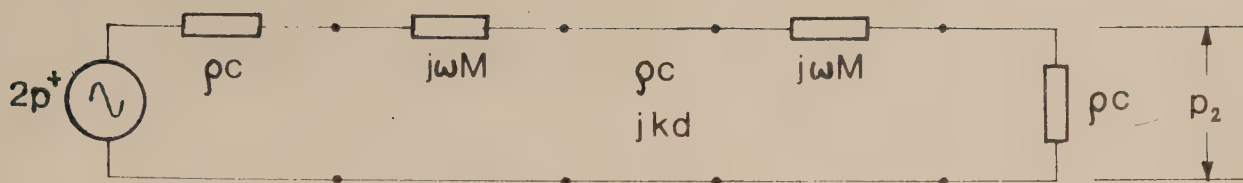
FIGURE 2.7.1. The driving field one-port and a single partition two-port.

Instead of using calculations with I_1^+ , I_1^- and I^+ and the boundary conditions, the transmission factor is simply obtained as the power in the ρc -termination, compared to the available power $p^{+2}/\rho c$ of the generator. We get the usual $I_1^+ = p^{+2}/\rho c$. The load for a (+)-wave is ρc , so we get $2p^{+2}/2 = p^{+2}$.

As both lines are matched in one end and have zero attenuation, they can be eliminated. Instead of waves a $2p^+$ -generator is connected to the two-port network of the partition. The generator still has the generator impedance ρc and the pressure ability $2p^+$.



a) Single partition.



b) Double partition with air space.

FIGURE 2.7.2. Transmission networks.

The transmission factor is defined as the first members below, leading to the model used here:

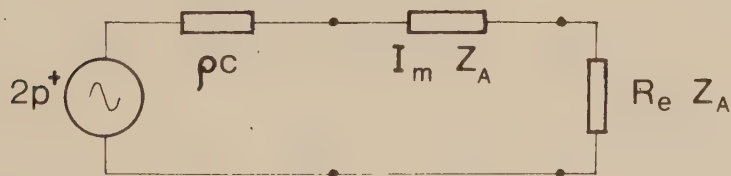
$$\tau = \frac{\left| \frac{I_2^+}{I_1^+} \right|}{\left| \frac{u_2}{p^+} \right|^2 / \rho c} = \frac{|u_2|^2 \rho c}{|p^+|^2 / \rho c} = \left| \frac{p_2}{p^+} \right|^2 \quad (1)$$

For the simple partition we get the velocity and the transmission factor according to the following expressions:

$$u_2 = \frac{2p^+}{2\rho c + j\omega M} \quad (2)$$

$$\tau = \frac{1}{1 + \left| \frac{j\omega M}{2\rho c} \right|^2} \quad (3)$$

Absorbents at walls can be dealt with in a similar way.



Z_A = impedance at the surface of the absorbent.

FIGURE 2.7.3. The absorption network.

We obtain the absorption factor α from the power in $\text{Re}(Z_A)$ compared to the available power.

$$\alpha = |u|^2 \operatorname{Re}(Z_A) / (p^{+2} / \rho c) \quad \quad \quad (4)$$

$$\alpha = \frac{4\rho c \operatorname{Re}(Z_A)}{(\rho c + \operatorname{Re}(Z_A))^2 + (\operatorname{Im}(Z_A))^2}$$

A "natural" absorbent frequently used is a free hanging fabric. We assume a flow resistance R and a weight M , which will give the following network:

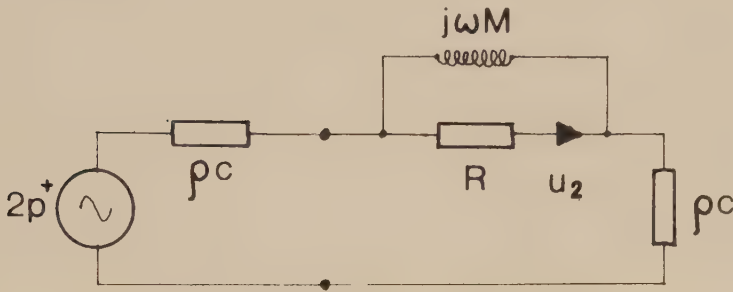


FIGURE 2.7.4. Free, thin absorbent network.

In this case the absorption is the power in R , so α is also influenced by the mass reactance:

$$\alpha = |u_2|^2 R / (p^{+2} / \rho c) \quad \quad \quad (5)$$

$$u_2 = \frac{2p^+ j\omega M / (j\omega M + R)}{2\rho c + \frac{1}{1/(j\omega M) + 1/R}}$$

u_2 will be small if $\omega M < R$. We see that lightweight fabrics are not very efficient with respect to absorption. For $R_s = 2\rho c$ and $\omega M \gg R$ we get

$$\alpha_{\max} = 1/2 \quad \quad \quad (6)$$

(α is sometimes defined as $1 - |B|^2$, where B is the reflexion factor. This is not valid for the free absorbent of the type shown above.)

The pressure generator can always be replaced by a velocity generator.

If the two-port network starts with parallel elements, it is convenient to replace the pressure generator by a velocity generator. As an example we choose a single leaf partition with resilient surfaces, e.g. thin absorbents with the admittance $v/\rho c$.

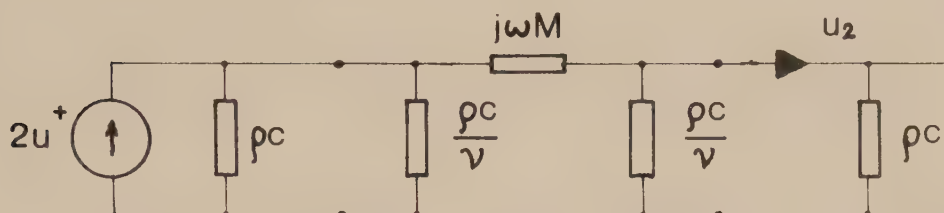


FIGURE 2.7.5. The network for a partition with resilient surfaces.

With a pressure generator the network would have been an unsymmetrical one. Now we can put ρc and $\rho c/v$ together.

$$\tau = |u_2/u^+|^2 \quad (7)$$

The first factor gives u_1 and the second factor the part u_2 of u_1 .

$$u_2 = u^+ \frac{1}{1 + \frac{j\omega M(1+v)}{2\rho c}} \cdot \frac{1}{1+v} \quad (8)$$

By choosing the suitable generator many calculations can be simplified as demonstrated above. This method will be used not only for transmission problems, but also for the transmission studies in Part 5. This leads to a new approach to transmission from a point source at a resilient surface.

As expected the field generators give the pressure $2p^+$ if the load impedance is ∞ (hard wall) and the velocity $2u^+$ if the load is perfectly resilient. The former case is usual for gases and the latter for fluids.

To give one more picture of the masslaw case we shall show a mechanical analogous system.

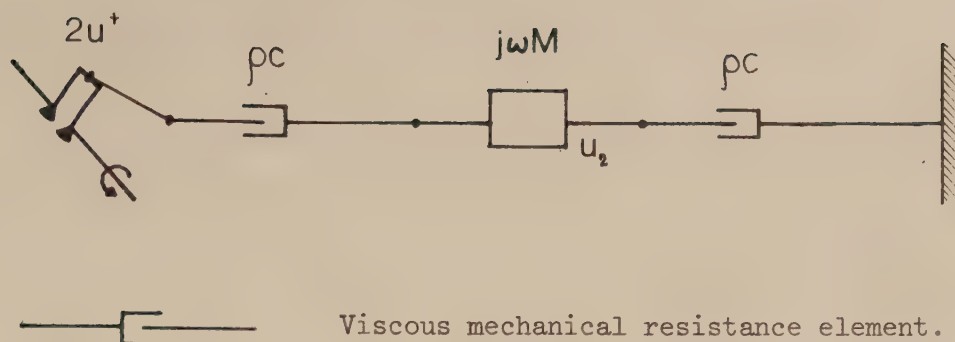


FIGURE 2.7.6. A mechanical system showing mass-law transmission.

We conclude that the driving field with a primary wave p^+ can be treated as a pressure generator with the pressure ability $2p^+$ or as a velocity generator with the velocity ability $2u^+$. In both cases the generator impedance is ρc for perpendicular waves.

The available power is $I^+ = p^{+2}/\rho c = u^{+2} \rho c$ and the transmission factor is the termination power compared to available power.

The cases demonstrated show that the concept of plus- and minus-waves is not always the best way to handle transmission and absorption problems. These acoustical applications are of great help, especially when the networks are formed to get analytical expressions for further computer calculations (Part 3).

2.8. The mean transmission and a new transmission factor

At oblique incidence the network for the infinite simple wall can be chosen according to FIG 2.8.1.

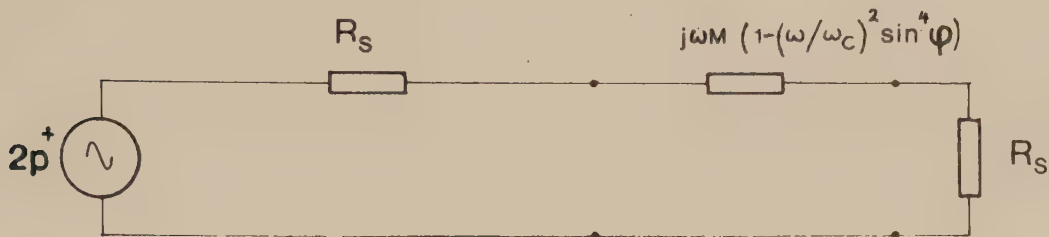


FIGURE 2.8.1. The oblique transmission.

With this choice $\vec{u}$ and $\vec{I}$ are perpendicular to the partition surface. The oblique transmission factor τ is usually defined from $\vec{I}_\phi^+$ in the wave direction:

$$\tau(\phi) = \frac{I_{2\phi}^+}{I_{1\phi}^+} \approx \frac{I_2^+(\phi)}{I_{1\phi}^+ \cos \phi} = \frac{|u_2(\phi)|^2 R_s}{(|p^+|^2 / \rho c) \cos \phi} \quad (1)$$

The "fiction" behind this is that an opening ($M = 0$), if $I_2^+ = I_{2\phi}^+ \cdot \cos \phi$, shall give $\tau(\phi) = 1$. We often use the mean transmission τ_s according to the following expression:

$$\tau_s = 2 \int_0^{\pi/2} \tau(\phi) \cdot \cos \phi \cdot \sin \phi \, d\phi \quad (2)$$

If the area is limited the radiation resistance $R_s \neq \rho c / \cos \phi$ and for the grazing case $R_s \neq \infty$, but $1/\cos \phi = \infty$ (cf. par. 2.4). In such cases the definition (1) is not very useful, as $\tau(\phi) \rightarrow \infty$, where $\phi \rightarrow 0$.

For partitions smaller than the wavelength, e.g. a window at low frequency, $R_s(\phi)$ is almost constant for varying ϕ . In such cases the definition in eq.(1) introduces a variation of τ although $I_2(\phi)$ is almost constant. This is possible as $\cos \phi$ comes in once more in the expression for I_2 :

$$I_2^+(\phi) = I_1^+(\phi) \tau(\phi) \cos \phi \quad (3)$$

For a finite partition calculation the product $\infty \cdot 0$, as $\phi \rightarrow \pi/2$, is very inconvenient, and I prefer to define a new transmission factor τ' :

$$\tau' = \frac{|u|^2 R_s}{p^2 / \rho c} \quad (4)$$

Instead of a weighting function $(\sin \phi \cos \phi)$ we only get a function $(\sin \phi)$:

$$\tau_s = 2 \int_0^{\pi/2} \tau(\phi) \sin \phi \, d\phi \quad (5)$$

We also define an insertion loss $IL(\phi)$ connected with $\tau'(\phi)$:

$$IL(\phi) = 10 \log(1/\tau'(\phi)) \quad (6)$$

The connection is the same as between $\tau(\phi)$ and $TL(\phi)$:

$$TL(\phi) = 10 \log(1/\tau(\phi)) \quad (7)$$

The idea behind τ' is that we have to distinguish between the factor $\cos \phi$ in the perpendicular component of $I_{1\phi}^+$ and the factor $1/\cos \phi$ in the radiation resistance. In practice $I_{1\phi}^+ \cos \phi$ and I_2^+ are calculated by pressure measurements. For grazing sound $I_{1\phi}^+ \cos \phi$ is zero ($\cos \phi = 0$), but I_2^+ is non-zero if $p_2 \neq 0$, which is always the case. We use the following connections for the measured p_1 and p_2 :

$$I_{1\phi}^+ \cos \phi = \frac{p_1^2}{\rho c} \cos \phi \quad (8)$$

$$I_{2\phi}^+ = \frac{p_2^2}{\rho c} \quad (9)$$

I discovered the need of τ' in the calculations on multiple leaf partitions as chainfilters [11]. In connection with traffic noise τ' and IL can be useful. In [12] it is shown how TL for traffic noise at windows varies with ϕ , but also that most of the variation is eliminated in cal-

culating the transmitted intensity I_2 (cf. eq.(7)). I_2 gives the indoor noise level (L_{ind}).

If the author had used IL instead of TL he might have got a description of the window, rather invariant of ϕ within a wide range. Then much of the tabelling of $TL(\phi)$ could have been eliminated, as we have the following connections for the outdoor and indoor levels:

$$IL = TL + 10 \log \cos \phi \quad (10)$$

Hence we obtain L_{ind} directly from IL, without $\cos \phi$. A is the absorption area of the room and S is the area of the window.

$$\begin{aligned} L_{ind} &= L_{out} - TL - 10 \log \cos \phi - 10 \log \frac{A}{4S} = \\ &= L_{out} - IL - 10 \log \frac{A}{4S} \end{aligned} \quad (11)$$

Finally we conclude that the window is more like a perfect reflector than an open hole seen from the driving field. The power load on the primary field is often negligible. Thus the pressure generator concept and the use of τ' suit many transmission problems better than the I^+ -open hole - $\tau(\phi)$ -idea.

The factor $A/(4S)$ will be further discussed in Part 4.

2.9. The finite radiation resistance

In par. 2.4 is shown that the radiation resistance is limited by $\approx 4/(kl)$ for $\phi = \pi/2$. We shall now study the grazing case and propose a simple radiation resistance to be used in practice.

From the result of par. 2.4 we can get an equivalent minimum angle ϕ_1 for radiation. The corresponding resistance is $\rho c / \cos \phi_1$ and we get:

$$\frac{1}{\cos \phi_1} = \frac{kl}{4} = \frac{\pi l}{2\lambda} \rightarrow \frac{\pi}{2} - \phi_1 \quad \psi_1 = \arcsin \frac{2\lambda}{\pi l} \approx \frac{2\lambda}{\pi l} \quad (1)$$

For grazing incidence we can imagine a transmission according to the following figure. The radiation is similar for a reverberant plate field at the critical frequency.

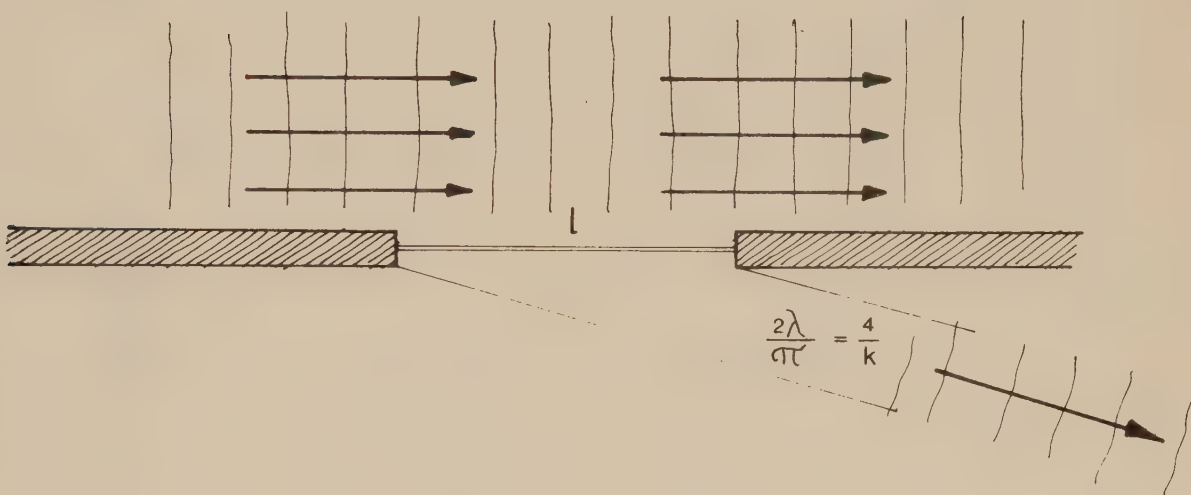


FIGURE 2.9.1. Radiation for grazing incident sound.

This figure is used in [11] and [13], where the radiation resistance was used in the following form as the grazing $R_s = kl/4$ was not deduced (G is an arbitrary constant):

$$R_s \approx \frac{\rho c}{\cos \phi + G/f} \quad (2)$$

The result of par. 2.4 will be used in TL-calculations as a general correction for $kl \gg 4$:

$$R_s \approx \frac{\rho c}{\cos \phi + 4/(kl)} \quad (3)$$

The minimum beam width for the grazing radiation is about the same as could be expected from the conphase narrow strip radiation in par. 2.4. When $kl > 1/4$ the radiation resistance is almost ρc , so the radiated beam does not diverge much. The following figure shows the radiation for $kl \gg 1/4$, $kl = 1/4$ and $kl < 1/4$.

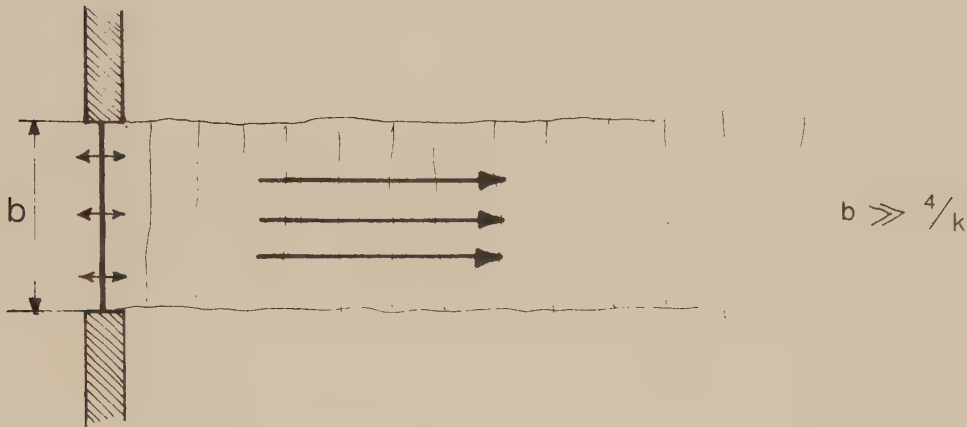


FIGURE 2.9.2.a. Broad strip, parallel field.

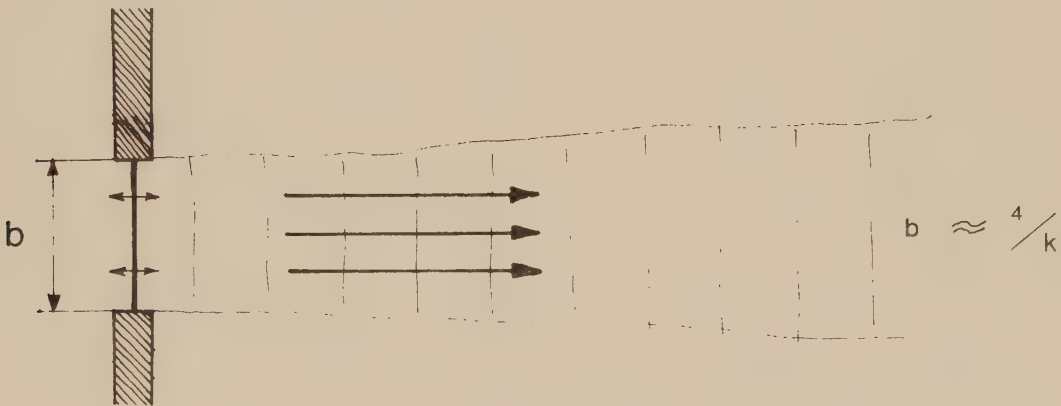


FIGURE 2.9.2.b. Narrow strip, almost parallel field. Small divergence.

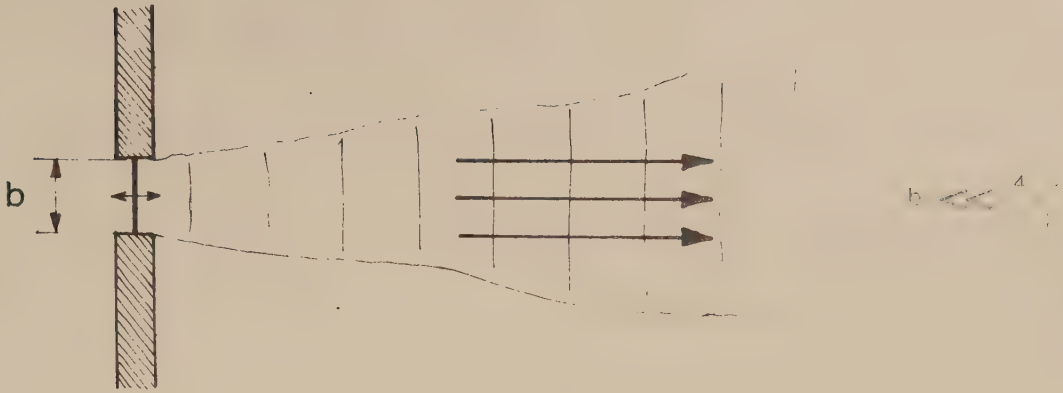


FIGURE 2.9.2.c. Very narrow strip, diverging field.

The results can also be interpreted as a general form of minimum beam width, valid for narrow strips and for bending wave sources. In the latter case we can use the idea also for a plate ab in all grazing directions. The reason is that if we put $a = 1$, the conditions above are valid if $b \gg 4/k$, but b does not have to be $\gg a$. We can get a flat beam also for $a \approx b$, as the beam cross section for grazing radiation is $b \cdot 4/k$. These considerations are very important for the simplifications made in this report.

2.10. The symmetrical partition

For a symmetrical double wall we get the following network:

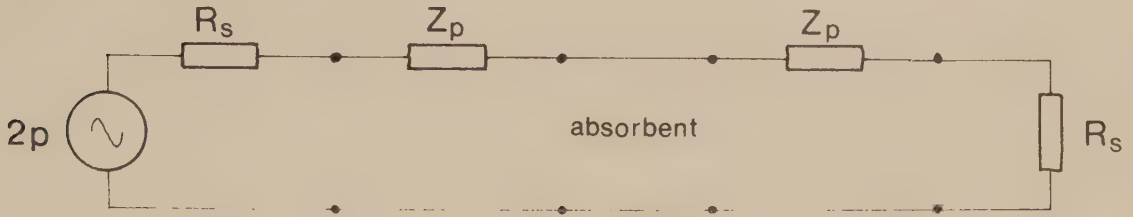


FIGURE 2.10.1. Network with absorbent in the double partition air space with the width d .

The "line" has a wave impedance Z_A according to the absorbent filling and a wave constant g . In this case we have a symmetrical two-port, which can be splitted in two unsymmetrical (image) two-ports [4] :

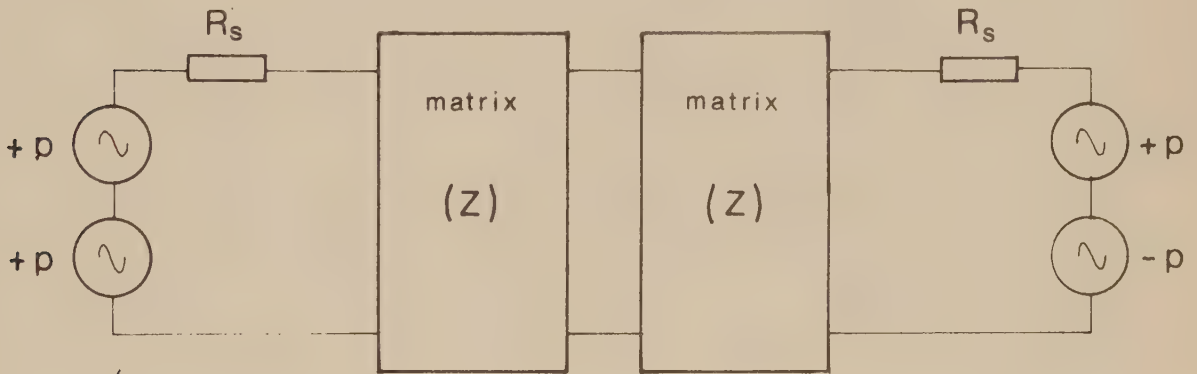


FIGURE 2.10.2. The symmetrical two-port splitted in two image two-ports.

The pressure is splitted in $p + p$ and $p + (-p)$ is added at the load side. This can be called an inphase-counterphase method, and if we use $+p_1 - +p_2$ and $+p_1 - -p_2$ we get only one of the two-ports in each generator combination.

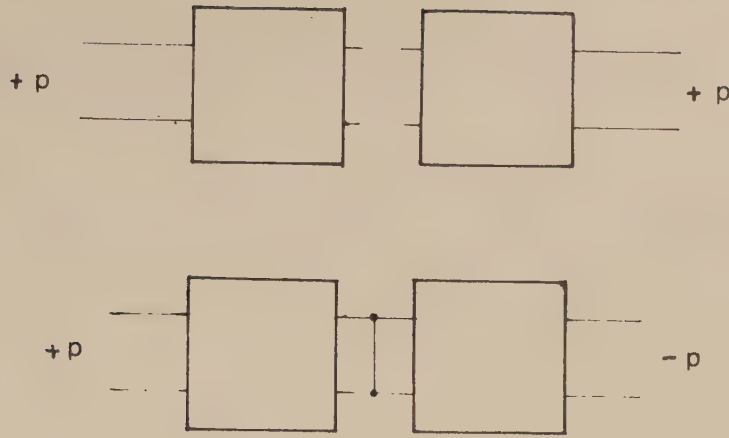


FIGURE 2.10.3. The inphase and counterphase give open and short circuit between the two-ports.

A combination of FIG 2.10.1 and 2.10.3 will give:

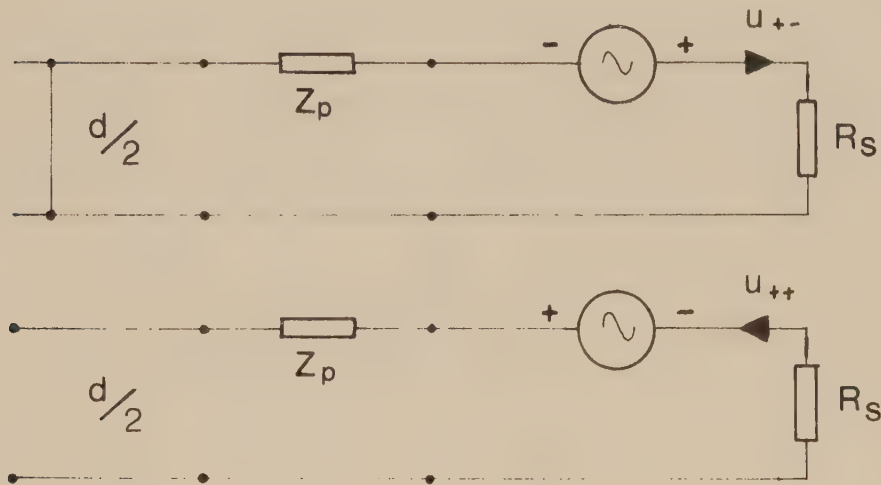


FIGURE 2.10.4. Resulting load on the load side generator.

The total velocity at the load is:

$$u = u_{+-} - u_{++} \quad (1)$$

With the simple expression for open and short circuited line we get the following connections:

$$Z_{\text{short}} = Z_A \tanh(gd/2) + Z_p + R_s \quad (2)$$

$$Z_{\text{open}} = Z_A \coth(gd/2) + Z_p + R_s \quad (3)$$

The superposition of the two velocities u_{+-} and u_{++} gives the solution:

$$u = \frac{p}{R_s + Z_p + Z_A \operatorname{tgh}(gd/2)} - \frac{p}{R_s + Z_p + \operatorname{coth}(gd/2)} \quad (4)$$

and

$$\tau = |u|^2 R_s / (p^2 / \rho c) \quad (5)$$

The sketched method is especially simple when computer calculations are used for the evaluation.

2.11. Radiation with losses in the air

The grazing radiation has been shown to give finite resistance for a limited plate to the infinite half-space. For grazing radiation from an infinite plate or from a finite plate to a room (wave guide) we must consider the losses in the air to get a limited radiation resistance for grazing radiation. With a loss factor η we get the following expressions for the plate vibration term m (cf. par. 2.6). The m -term is of $(\cos \cdot \cos)$ -type to fit the airfield boundaries.

$$k_z^2 + k_{mx}^2 + k_{my}^2 = k_z^2 + k_m^2 = \bar{k}^2 = k^2(1 - j\eta) \quad (1)$$

$$k_z = k \sqrt{1 - j\eta - (k_m/k)^2} = k \sqrt{1 - \xi^2 - j\eta} \quad (2)$$

$$k_m/k = \xi \quad (3)$$

$$k_z = k \sqrt{\cos^2 \phi - j\eta} \quad \text{if } \xi < 1 \quad (4)$$

$$Z = \frac{\rho c}{k_z/k} = \frac{\rho c}{\sqrt{1 - \xi^2 - j\eta}} \quad (5)$$

"Wave guide":

$$p(x, y, z) = \sum \sum u_{pm} P_m(x, y) \frac{\rho c(1 - j\eta)}{\sqrt{1 - \xi^2 - j\eta}} e^{-jk_{mz}z} \quad (6)$$

Room:

$$p_1(x, y, z) = \sum \sum -j u_{pm} P_m(x, y) \frac{\rho c}{\sqrt{1 - \xi^2 - j\eta}} \cdot \frac{\cos(k_{mz}(1 - x))}{\sin(k_{mz}l)} \quad (7)$$

For $\cos^2 \phi \gg \eta$ we get

$$k_z \approx k \left(\cos \phi - \frac{j\eta}{2 \cos \phi} \right) \quad (8)$$

This expression shows how the attenuation $k\eta/(2 \cos \phi)$ in the z -direction increases for oblique mode radiation although the wavenumber decreases. This can also be expressed as a group velocity $c_0 \cos \phi$ and a phase velocity $c_0/\cos \phi$. Evidently the losses affect the longer "group path" $l/\cos \phi$.

The standing wave resistance at resonance can be developed from eq.(7) with k_z according to eq.(8). We get the following deduction:

$$jk_z l = gl = \alpha + j\beta \quad (9)$$

$$\alpha = \frac{lk\eta/2}{\cos \phi} \quad ; \quad \beta = lk \cos \phi \quad (10)$$

$$\begin{aligned} \frac{Z_s}{\rho c} &= \frac{\bar{k}}{k_z} \cdot \coth(gl) = \frac{\bar{k}}{k_z} \cdot \frac{\cosh(\alpha + j\beta)}{\sinh(\alpha + j\beta)} = \\ &= \frac{\bar{k}}{k_z} \cdot \frac{\cosh \alpha \cos \beta + j \sinh \alpha \sin \beta}{\sinh \alpha \cos \beta + j \cosh \alpha \sin \beta} \end{aligned} \quad (11)$$

Near resonance we have $\beta = n\pi + \Delta\beta$ and $k = k_r + \Delta k$. Hence we get:

$$\begin{aligned} \frac{Z_s}{\rho c} &\approx \frac{k}{k_z} \cdot \frac{1}{\alpha + j\Delta\beta} \approx \frac{1}{\cos \phi} \cdot \frac{1/(lk)}{\frac{\eta}{2 \cos \phi} + j \frac{\Delta k}{k} \cos \phi} \approx \\ &\approx \frac{1}{lk} \cdot \frac{1}{\eta/2 + j(\Delta k/k) \cos^2 \phi} \end{aligned} \quad (12)$$

The peak resistance is $\eta/2$ as it was for a plane piston, but the peak width (3 db) is increased from $k\eta$ to the following expression:

$$(\text{Half power width}) = \frac{k\eta}{\cos^2 \phi} \quad (13)$$

2.12. Grazing radiation with losses in the air

If $\kappa_m = k(\cos \phi = 0)$ we get:

$$g = jk \sqrt{-j\eta} = k(1 + j) \sqrt{\frac{\eta}{2}} \quad (1)$$

$$Z_\infty = \frac{\rho c}{\sqrt{-j\eta}} = \frac{1 + j}{\sqrt{2\eta}} \rho c \quad (2)$$

Wave guide:

$$p_m(z) = u_{mp} \cdot Z_o \cdot e^{-gz} \quad (3)$$

Room:

$$p_{ml}(z) = u_{mp} \cdot Z_o \cdot \frac{\cosh((1 + j) k \sqrt{\eta/2} (1 - z))}{\sinh((1 + j) k \sqrt{\eta/2} l)} \quad (4)$$

$$Z_1 = Z_o \coth((1 + j) k \sqrt{\eta/2} l) \quad (5)$$

The R_s -limitation can be translated into a " ϕ -limitation", but the reason is not lack of standing waves or modes of grazing type. We put

$$\rho c / \cos \phi_g = \rho c / \sqrt{2\eta} \quad (6)$$

$$\phi_g = \arccos \sqrt{2\eta} \approx \pi/2 - \sqrt{2\eta} \quad (7)$$

As an example we choose $\eta = 3 \cdot 10^{-4}$ and $\phi_g = 2^\circ$. The attenuation of the field in the z -direction is:

$$k \sqrt{\eta/2} \text{ neper/m} \quad (8)$$

$$8.7k \sqrt{\eta/2} \text{ dB/m} \quad (9)$$

The plane wave attenuation is only $k\eta/2$ neper/m or $4.35k\eta$ dB/m. For the

example above this will be 0.07 dB/m. From eq.(9) we get 7 dB/m at 4 kHz, which is of a different order of magnitude.

2.13. Concluding remarks to Part 2

The methods introduced in this part concerning radiation and transmission networks can be regarded as alternative models for calculations. I claim that the manipulations can be improved and speeded up by the presented methods, but the opinion about this might depend on the reader's technical background.

The choice of the first paragraphs also depends on my experience that the Fourier spatial transforms are not as familiar as the usual spectral analysis of sound. This is particularly true for people graduated in electrical engineering and I hope that the demonstrations of simple cases will improve the comprehension of the following parts.

I also hope that the driving field one-port, the simplified radiation resistance and the transmission factor τ' will be discussed and used.

PART 3

MODIFICATION OF WAVE EQUATIONS, ABSORBENTS, NARROW SPACE WAVES
AND MULTIPLE PARTITIONS

3.1. Introduction

One-dimensional waves are fairly easy to handle and have applications to noise control. The starting point is often the knowledge of a standard wave type and the solution is a modification of that wave. Some methods applicable to plane waves are brought up. The double leaf with a narrow air space with flow resistance is an interesting element of the multiple partition and will therefore be subjected to some analysis.

Computations are performed for a special window with one very narrow space.

Some interest is also devoted to absorption and the question of the statistical absorption factor being larger than 1.

3.2. Plane wave modification and ideal absorbent

The ideal absorbent is a rigid structure giving a flow resistance and can often be used with sufficient accuracy as an element in absorption calculations. It is also used as a component in TL-calculations for multiple leaf partitions. Usually it is convenient to start with the well-known wave equation for air and rewrite it in order to find the modifications giving new wave parameters. This is a common method for circuits and analogous networks are therefore shown.

The brief comments on absorption are motivated by the use of simple networks and modification methods and also by the further use in multiple walls and in Part 5.

For a plane wave we can write the wave impedance Z_0 and the transmission constant g as follows. The purpose is to use the factors z_1 and z_c further on. In air we have one density factor and one compliance factor, which form z_0 and g respectively:

$$Z_0 = \rho c = \sqrt{j\omega\rho \frac{\rho c^2}{j\omega}} = \sqrt{z_1 z_c} = \sqrt{z_1 / y_c} \quad (1)$$

The transmission constant is expressed in the same factors z_1 and z_c :

$$g = j\omega/c = \sqrt{j\omega\rho / \frac{\rho c^2}{j\omega}} = \sqrt{z_1 / z_c} = \sqrt{z_1 y_c} \quad (2)$$

z_1 can be interpreted as an "along" property and z_c as a "cross" property. The "along" impedance z_1 and the "cross" impedance z_c are illustrated by the following line and network.



FIGURE 3.2.1. Analogous line and cross link for modification of Z_0 and g .

For an "ideal" absorbent, which is a rigid structure with flow resistance σ and only affecting the compression (isentropic to isothermal) we obtain:

$$z_1 = j\omega\rho + \sigma = j\omega\rho \left(1 + \frac{\sigma}{\rho c jk} \right) \quad (3)$$

The flow resistance is added in z_1 . The structure affects the heat capacity and we get:

$$z_c = p_{\text{atm}} \mu / j\omega = \frac{\rho c}{jk} \cdot \frac{\mu}{\gamma} \quad (4)$$

$\gamma = c_p/c_v$, the ratio of specific heats, = 1.4 for air. μ is the polytropic exponent for compression types between 1 and γ , if it is neither isentropic nor isothermal, see [17]. The flow resistance σ is normalized, $r = \sigma/\rho c$. σ and r for two Swedish mineral fibre types are shown in FIG 3.2.2.

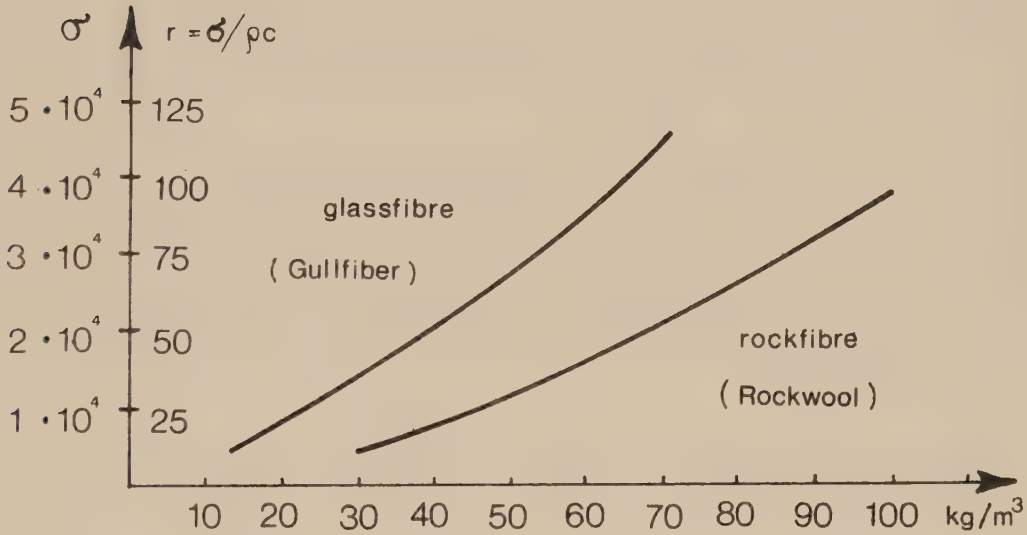


FIGURE 3.2.2. Specific flow resistance, from [18].

We get the following expression for the wave in the absorbent (Z_A is the wave impedance, g_A is the transmission constant):

$$Z_A = \sqrt{z_1 z_c} = \rho c \sqrt{\frac{\mu}{\gamma} \left(1 + \frac{r}{jk} \right)} ; \quad Y_A = 1/Z_A \quad (5)$$

$$g_A = jk \sqrt{\frac{\gamma}{\mu} \left(1 + \frac{r}{jk} \right)} \quad (6)$$

For $r \gg k$ and $\mu = \gamma$ we get some simplified connections:

$$Z_A = \rho c(1 - j) \sqrt{r/(2k)} \quad (7)$$

$$g_A = k(1 + j) \sqrt{r/(2k)} \quad (8)$$

These wave parameters are of the same type as for the RC-line in telephony.

With $\gamma = \mu$ we get the reflexion factor δ and the relative, effective impedance ζ_e :

$$\zeta_e = \frac{1 + \delta}{1 - \delta} = \frac{\sqrt{1 + r/(jk)} \cos \phi}{\sqrt{1 - \sin^2 \phi / (1 + (r/jk))}} \quad (9)$$

$$\delta = \frac{\zeta_e - 1}{\zeta_e + 1}$$

If $k \gg r$, $\zeta_e \approx 1$ and $\delta \approx 0$. If $r \gg k$ the denominator ≈ 1 and ζ_e is called a point impedance, as only the point has importance, not the wave with its direction in the absorbent. The concept will be used in Part 5. For a grass ground looked upon as an absorbent we normally have $r \gg k$.

3.3. The thin absorbent

For normal incidence a thin absorbent on a hard surface gives the following network:

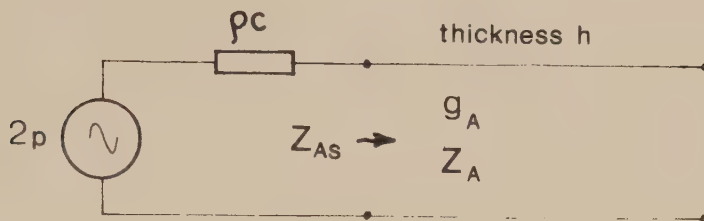


FIGURE 3.3.1. Absorbent network at hard boundary $\equiv$ open end.

This type of impedance is often expanded in its first terms of Z_{AS} . This is of interest to the low frequencies and especially to the grass ground "absorbent".

The standing wave gives Z_{AS} , the "surface" impedance, see [19]. $Z_S = 1/Y_S$, h is the thickness, Z_A is the wave impedance and g is the wave constant.

$$Z_{AS} = Z_A \coth(gh) = Z_A \left(\frac{1}{gh} + \frac{gh}{3} + \dots \right) \quad (1)$$

$$Y_{AS} = Y_A \tanh(gh) = Y_A \left(gh + \frac{(gh)^3}{3} + \dots \right) \quad (2)$$

For thin absorbents we take the first terms and from (1) we get:

$$\begin{aligned} Z_{AS} &\approx \frac{Z_A}{gh} + \frac{gh}{3} Z_A = \frac{z_c}{h} + \frac{z_l}{3} h = \rho c \left(\frac{\mu}{jkh\gamma} + \frac{rh}{3} + \frac{jkh}{3} \right) \approx \\ &\approx \rho c \left(\frac{\mu}{jkh\gamma} + \frac{rh}{3} \right) \end{aligned} \quad (3)$$

From (2) we obtain

$$Y_{AS} \approx \frac{1}{\rho c} jkh\gamma/\mu \left(1 + \frac{(hk)^2}{3\gamma/\mu} (1 + r/jk) \right) \quad (4)$$

and from (1):

$$Y_{AS} = 1/Z_{AS} = \frac{\left(\frac{rh}{3} + \frac{j\mu}{kh\gamma}\right)}{\left(\left(\frac{\mu}{kh\gamma}\right)^2 + \left(\frac{rh}{3}\right)^2\right)\rho c} \quad (5)$$

(3) is often used. (5) is usually more convenient than (4).

For $|g_A| \gg k$ the air vibration u_s is perpendicular also for oblique incidence to the surface. We therefore get the following network:

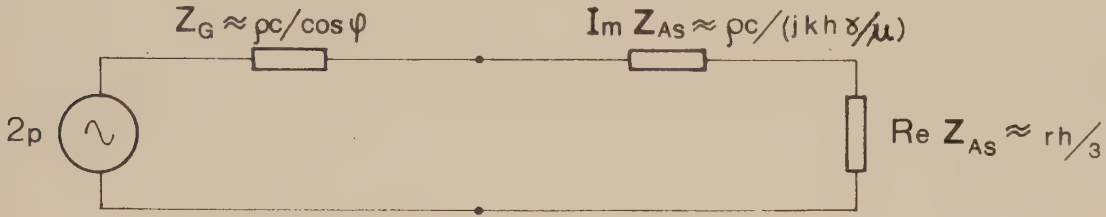


FIGURE 3.3.2. Thin absorbent at oblique incidence.

The absorption factor $\alpha(\phi)$ is:

$$\alpha(\phi) = \frac{|u|^2 \operatorname{Re}(Z_{AS})}{(p^2/\rho c) \cos \phi} = \frac{4\operatorname{Re}(Z_{AS})/(\rho c \cdot \cos \phi)}{\left(\frac{1}{\cos \phi} + \frac{\operatorname{Re}(Z_{AS})}{\rho c}\right)^2 + \left(\frac{\operatorname{Im}(Z_{AS})}{\rho c}\right)^2} \quad (6)$$

For a thin absorbent we use (3) and obtain:

$$\alpha(\phi) = \frac{4rh/(3 \cos \phi)}{\left(\frac{1}{\cos \phi} + \frac{rh}{3}\right)^2 + \left(\frac{\mu}{kh\gamma}\right)^2} \quad (7)$$

3.4. The mean absorption and a new absorption factor $\alpha'(\phi)$

The mean (statistical) absorption in a "diffuse field" is usually defined as α_S :

$$\alpha_S = \int_0^{\pi/2} 2\alpha(\phi) \cdot \cos \phi \cdot \sin \phi \, d\phi \quad (1)$$

If $R_G \neq \rho c / \cos \phi$ the result will be that $\alpha(\phi) \rightarrow \infty$ for $\phi = \pi/2$. We now face the problem of $\infty \cdot 0$ just as we did for $\tau(\phi)$ in par. 2.8 (cf. [16]).

In some cases a new factor $\alpha'(\phi)$ similar to $\tau'(\phi)$ (see eq.(2.8.6)) can be more convenient:

$$\alpha'(\phi) = \frac{|u|^2 \operatorname{Re} Z_{AS}}{p^2 / \rho c} \quad (2)$$

and

$$\alpha_S = \int_0^{\pi/2} 2\alpha'(\phi) \sin \phi \, d\phi \quad (3)$$

With this definition it is perhaps easier to see why α_S can be > 1 [20]. $\alpha'(\phi)$ is absorbed power per area unit of the absorbent (not of the wave as is $\alpha(\phi)$). The integral of the weighting function is:

$$\int_0^{\pi/2} 2 \sin \phi \, d\phi = 2 \quad (4)$$

If $|Z_{AS}| \gg \rho c$ (FIG 3.3.1) there is a tendency for u being independent of ϕ , so we directly get $\alpha_S \approx 2\alpha'(0) = 2\alpha(0)$. This is typical for small $\alpha(0)$ -values (cf. [15]).

If the absorbent sample area is small there is a similar tendency also for larger $\alpha(0)$ -values.

If the absorbent is thick and $r \ll k$, the surface does not act as a point impedance and the particle velocity is not perpendicular to the surface. If we use an ideal absorbent we get:

$$Z_{AS} = R_{AS} = \rho c / \cos \phi \quad (5)$$

With a field impedance $\rho c / \cos \phi$ we obtain:

$$\alpha'(\phi) = \frac{4 / \cos \phi}{(1 / \cos \phi + 1 / \cos \phi)^2} = \cos \phi \quad (6)$$

We shall now use the idea of R_S given in eqs.(2.4.5) and (2.9.2):

$$R_S \approx \rho c / (\cos \phi + 4 / (kl)) \quad (7)$$

$$R_{AS} = R_G = R_S \quad (8)$$

This result is the approximative $\alpha'(\phi)$:

$$\alpha'(\phi) = \cos \phi + 4 / (kl) \quad (9)$$

If $4 / (kl)$ is omitted as in eq.(5), $\alpha_S = 1$. Now we shall get an extra increment

$$\alpha_S \approx 1 + \frac{4\pi}{kl} \quad (10)$$

This is a simplified model, but it gives an explanation to the often measured $\alpha_S > 1$.

Reverberant rooms are equipped with diffusors or irregularities on the walls. The aim of this facility is to get the same reverberation of all modes. Then all models must act as being influenced by α_S instead of $\alpha(\phi_n)$. ϕ_n is the "angle of incidence" for the mode. No mode will have $Z_G = 4 / (kl)$ from a specific angle $\phi_n = \pi/2$. The variation of k-numbers along the absorbent area will make an unlimited field impedance impossible. This can increase α_S compared to the sketched model. We plan to continue with a mode varying in k_z (k normal to the surface) in different points. I suspect that the k_z -variation within a mode will act as if l were less than the typical dimension of the sample. We will get more of the shown effect that $\alpha_S \rightarrow 2\alpha(0)$, also for larger $\alpha(0)$ -values.

3.5. Waves in tubes

In this paragraph the use of z'_1 and z'_c defined below for narrow tube waves, is shown as a background to the double leaf partition wave. The examples are well-known and given just to show further possible modifications of the simple wave equations.

In a narrow tube ($< \lambda/4$) the wave is here referred to as one-dimensional and not as a plane wave. For the cross section S we use the acoustical impedance Z' instead of the specific impedance. In analogy with par. 3.2 we get the following expressions with perfectly hard and rigid tube walls. (') denotes volume velocity/pressure impedance:

$$Z'_o = \frac{\rho c}{S} = \sqrt{\frac{j\omega\rho}{S} \cdot \frac{\rho c^2}{j\omega S}} = \sqrt{z'_1 \cdot z'_c} \quad (1)$$

The wave constant or wavenumber will be the same as in a plane wave:

$$g' = jk = \sqrt{z'_1/z'_c} = \sqrt{z'_1 \cdot y'_c} \quad (2)$$

If the walls are resilient y'_c is increased by $y'_{c,incr}$. The walls with the perimeter P have the specific admittance $Y_w = v/\rho c$, which is added to y'_c :

$$y'_{c,incr} = Y_w \cdot P = (v/\rho c) P \quad (3)$$

With $\mu = \gamma$ we get for the new wave:

$$z'_1 = j\omega\rho/S = jk\rho c/S \quad (4)$$

$$y'_c = \frac{1}{\rho c} (jkS + vP) \quad (5)$$

For the one-dimensional tubewave we get the following acoustic wave impedance Z'_T from z'_c and y'_c above:

$$Z'_T = \rho c \sqrt{\frac{jk/S}{jkS + vP}} = \frac{\rho c/S}{\sqrt{1 + \frac{v}{jk} \cdot \frac{P}{S}}} \quad (6)$$

The propagation constant is also obtained in the same way:

$$g_T' = jk \sqrt{1 + \frac{vP}{jkS}} \quad (7)$$

We shall now regard v as determined by the strain. For a circular tube the strain in the wall is determined from Young's modulus E and the contraction μ . The thickness of the wall h can also be the average thickness of a helical steel reinforcement of plastic or rubber tubes. With the diameter D we obtain:

$$Y_W = \frac{(D/2)^2}{hE(1 - \mu^2)} \quad j\omega = \frac{1}{\rho c} \quad \frac{D^2 jk \rho c^2}{4\rho hE(1 - \mu^2)} = \frac{v}{\rho c} \quad (8)$$

$$P/S = 4/D \quad (9)$$

$$k_T' = k \sqrt{1 + \frac{D \rho c^2}{E(1 - \mu^2)}} \quad (10)$$

As Y_W is here a pure compliance, ω vanishes below the root.

For a fluid $k' = k_f'$ can be rather large, but k_f' can be lowered in such a degree that $k_f' < k_o$. The radiation to the air (k_o) is diminished (below critical coincidence). Example: Water: $c \approx 1500$ m/s, air: $c_o = 340$ m/s, $D = 0.1$, $h = 10^{-4}$, steel: $E = 2.2$ GN/m².

$$c' = \frac{c}{\sqrt{1 + 11}} \approx 430 \text{ m/s} \quad (11)$$

The speed c' is also the speed of pulse waves in blood veins (Moens Korteweg's equation, [21]).

An absorbent lining gives the following equation (with Y_W from eq.(3.3.4):

$$k_T' = k \sqrt{1 + \frac{hYP}{S} \left(1 + \frac{(hk)^2}{3Y} \left(1 + \frac{r}{jk} \right) \right)} \quad (12)$$

(cf. [16].)

The attenuation will be $\text{Im}k' = b_T'^2$ and if $h\gamma P < S$ and $r > k$ we get

$$b' = \frac{k}{2} \cdot \frac{h\gamma P}{S} \cdot \frac{(hk)^2}{3\gamma} \cdot \frac{r}{k} = \frac{1}{2} \cdot \frac{h^3 r}{3} \cdot \frac{P}{S} \cdot k^2 \quad (13)$$

We shall compare with a direct power model. If Z_{AS} is fairly high a normal incidence pressure at the surface is doubled (and gives α_0). Here the wave is not normal, but α_0 will be used according to the following assumptions. Accepted power per area unit in the absorbent is expressed in the incident wave p^+ and the tubewave p . I^+ defines α_0 and we get the α_0 -influence in the tubewave:

$$I_\alpha = \alpha_0 I_1^+ = \alpha_0 \frac{p^{+2}}{\rho c} = \frac{\alpha_0}{4} \cdot \frac{(2p^+)^2}{\rho c} = \frac{\alpha_0}{4} \cdot \frac{p^2}{\rho c} = \frac{\alpha_0}{4} I_{\text{tube}} \quad (14)$$

The wave has the intensity I_{tube} and we formulate the power connections:

$$\Pi = I_{\text{tube}} \cdot S \quad (15)$$

$$-\frac{\partial \Pi}{\partial x} = I_{\text{tube}} \frac{\alpha_0}{4} \cdot P \quad (16)$$

Additional comments upon α_0 could be mentioned. α_0 can be used for this one-dimensional wave as the "grazing" sound does not guide the vibration in its direction in a narrow tube. The one-dimensional wave can only be used in tubes or spaces less than the wavelength in cross measure (see also narrow space waves). We solve the "power wave":

$$\frac{\partial \Pi(x)}{\partial x} = -\frac{\alpha_0 P}{4S} \Pi(x) \quad (17)$$

$$\Pi(x) = \Pi(0) \cdot \exp\left(-\frac{\alpha_0 P}{4S} x\right) \quad (18)$$

$$2b'_\alpha = \frac{\alpha_0}{4} \cdot \frac{P}{S} \quad (19)$$

For a thin absorbent we get α_0 from par. 3.3:

$$\alpha_0 \approx \frac{4hr/3}{\left|\frac{1}{kh}\right|^2} = \frac{4k^2 h^3 r}{3} \quad (20)$$

Insertion of this expression in (19) gives:

$$2b'_\alpha = \frac{P}{S} \cdot \frac{k_h^2 r^3}{3} \quad (21)$$

With the approximation in k' we arrived at the same result as with the simple power model, both giving the attenuation B_{dB} :

$$B_{dB} = \frac{P}{S} \cdot \frac{\alpha_o}{4} \cdot 4.35 = \frac{P}{S} \cdot 1.1 \cdot \alpha_o \quad (22)$$

The simple power model is evidently useful and the assumptions of $\alpha_o/4$ correct (cf. [16]).

Eq.(22) can be used also for flat channels with $S/P = d$. d is the distance between the linings of the long sides of the cross section. The results shown here can also be used for waves in multiple partition spaces.

3.6. The impedance admittance of a narrow space between two leaves

The spaces in double walls are treated in different ways in literature. For the narrow space we can get a solution also with absorbent filling. We shall avoid the reflexion model, which is unphysical, as waves narrower than λ cannot exist (cf. Part 2, grazing radiation).

To obtain the solutions we use the components of par. 3.5. We add the volume flow from the difference between the velocities of the two leaves.

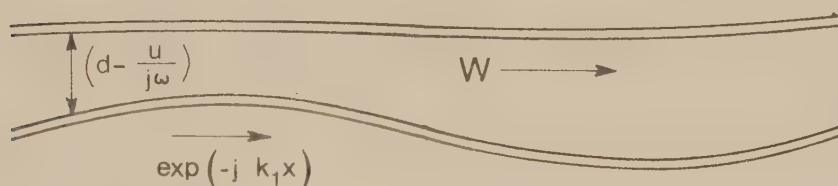


FIGURE 3.6.1. Forced motion on a space.

We obtain an inhomogeneous equation from the forcing term u , giving an increment in the continuity equation.

$$\text{Euler:} \quad -jk_1 p = W z'_1 \quad (1)$$

$$\text{Continuity:} \quad -jk_1 W + u = p y'_c$$

For the width variation u is added. With $u = 0$ we get the usual wave equation in air:

$$k'^2 = -z'_1 y'_c \quad (2)$$

With u we can solve a space impedance for forced motion: $Z_f = 1/Y_f = p/u$, from (1) and (2):

$$\begin{aligned} Y_f &= y'_c + jk_1 W/p = y'_c - k_1^2/z'_1 = -(k'^2 - k_1^2)/z'_1 = \\ &= y'_c (1 - (k_1/k')^2) \end{aligned} \quad (3)$$

If $k_1 = k'$ follows that $Y_f = 0$ and the space is non-resilient. This can

be regarded as a sonic boom effect. The field is forced at its own free wave velocity.

The analogy with the Cremer coincidence is obvious. At coincidence the leaf impedance Z_s is zero. However, it is unlikely that $k' < k$ if the forced k_1 comes from sound waves. This gives $k_1 = k \sin \phi \leq k$ and space wave coincidence is therefore not possible. With ideal absorbent filling z'_1 and y'_c are as follows:

$$z'_1 = (j\omega\rho + \sigma)/d = jk\rho c(1 + r/(jk))/d \quad (4)$$

$$y'_c = \frac{j\omega dy}{\rho c^2 \mu} = \frac{jkdy}{\rho c \mu} \quad (5)$$

The space admittance Y_f is:

$$Y_f = \frac{jk d}{\rho c} \left(\frac{\gamma}{\mu} + \frac{(k_1/k)^2}{1 + r/(jk)} \right) \quad (6)$$

The flow resistance r will give a pure, reactive Y_f both for $r = 0$ and $r \rightarrow \infty$. For high values of r , however, Y_f does not change with (k_1/k) , as Y_f does for $r = 0$. Other methods of inserting losses in the air space are discussed in [24].

From the expressions above we conclude that, probably also with absorbents, there can often be a deep resonance dip in a TL-curve at the resonance frequency for a double partition.

For $\mu = \gamma$ and $r = 0$ we get ($k_1 = k \sin \phi$):

$$Y_f = \frac{jk d}{\rho c} \cos^2 \phi \quad (7)$$

This is the same result as the reflexion method gives in [25]. Eq.(7) here was obtained with a one-dimensional wave, so the reflexions are a "second hand" picture of the behaviour of narrow air space.

The reflexion method uses plane waves of much less width than the wavelength. From the laws of wave propagation we realize that this is not

permitted. It only gives the correct result when $1 - (k_1/k)^2 = \cos^2 \phi$, the lossless case. However, such a solution cannot be used to determine how a resistance in the space affects the space admittance.

We shall make computations later for very narrow air spaces. The well-known resistance for air flow in a flat channel is therefore shown here. For the parabolic flow, with the viscosity of air $\nu \approx 18 \cdot 10^{-6} \text{ kg/m}^2\text{s}$, we get:

$$\sigma = 12\nu^2/d^2 \approx 0.22 \cdot 10^{-3}/d^2 \quad (8)$$

$$r = \sigma/\rho c \approx 0.5/(d \cdot 10^3)^2 \quad (9)$$

$$z_1' \approx \frac{jk\rho c}{d} \left(1 + \frac{1}{jk} \cdot \frac{0.5}{(10^3 d)^2} \right) \quad (10)$$

If $d < 10^{-3}/\sqrt{2k} = 10^{-3} \cdot \sqrt{27/f}$ r will be of great importance, but also for smaller values of r a considerable effect can be noticed.

In fact it is possible that the narrow space resistance is often better matched to jk than an absorbent filling, which gives too high a value of r . This results in almost purely reactive Y . Experiments are planned with absorbent fillings with very low flow resistance, to study the effect of an optimal loss angle of $(1 + r/jk)$.

Finally a TL-curve of a two-pane construction with $d < 1 \text{ mm}$ is shown in FIG 3.6.2. The usual resonance formula is:

$$f_0 = \frac{c}{2\pi} \sqrt{2\rho}/\sqrt{Md} \approx 85/\sqrt{Md} \quad (11)$$

For the window element in question we get $f_0 \approx 1 \text{ kHz}$.

There is no sign of the resonance in the TL-curve. One reason can be the complex narrow space admittance, which eliminates the resonance dip expected in the TL-curve.

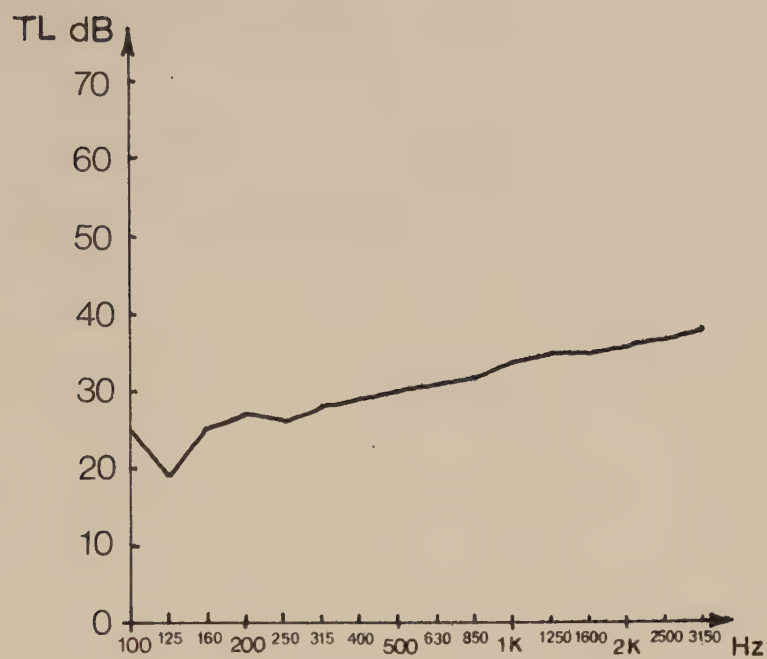


FIGURE 3.6.2. Transmission loss of double glazing with 0.5 mm air space.

3.7. The double partition without losses

The peaks of transmission are difficult to show analytically in the common case. If the partition is symmetrical and lossless there is a classical solution given in [22] for the transmission factor τ . We take a new interest in this solution for small air spaces with a high resonance frequency.

$$(1/\tau) = 1 + 4a^2(\cos \beta - a \sin \beta)^2 \quad (1)$$

a is the leaf reactance with $\kappa^4 = \omega^2 M/B$:

$$a = \frac{\omega M}{2\rho c} (1 - (k_1/\kappa)^4) \quad (2)$$

β is a phasenumber of the space:

$$\beta = 2kd(1 - (k_1/k)^2) \quad (3)$$

For a driving wave incident at the angle ϕ we get k_1 :

$$k_1 = k \sin \phi \quad (4)$$

This simple form makes it easy to find the transmission peaks $\tau = 1$, which are at hand if $a = 0$ or if the bracket in (1) is zero. $a = 0$ can be interpreted as a parallell (inphase) vibration of the leaves. The other solution is a counterphase, dilatational vibration, which will occur at a "dilatational frequency" f_d (cf. layered partitions, [26], p 212 and floating floors, [27]):

$$\cot(kd \cos \phi) = \frac{\omega M}{2\rho c} (1 - (f/f_c)^2 \sin^4 \phi) \cos \phi \quad (5)$$

For a small space d we can simplify:

$$\cot x \approx 1/x \quad (6)$$

The resonance frequency f_0 is

$$(2\pi f_o)^2 = \omega_o^2 = 2\rho c^2/(Md) \quad (7)$$

$$f_d^2 = \frac{f_c^2}{\sin^4 \phi} \left[\frac{1}{2} \pm \sqrt{\frac{1}{4} - \left(\frac{f_o \sin^2 \phi}{f_c \cos \phi} \right)^2} \right] \quad (8)$$

f_o is the resonance frequency and f_c the critical frequency.

If $f_o \sin^2 \phi \ll f_c \cos \phi$ the expansion of the root gives the solutions

$$f_{d1}^2 = f_o^2 / \cos^2 \phi \quad (9)$$

$$f_{d2}^2 = f_c^2 / \sin^4 \phi - f_o^2 / \cos^2 \phi \quad (10)$$

If $2f_o \sin^2 \phi = f_c \cos \phi$ there will be only one root, a double root:

$$f_d = f_c / (\sqrt{2} \sin^2 \phi) \quad (11)$$

If $2f_o \sin^2 \phi > f_c \cos \phi$ there are no real solutions and hence no dilatational "coincidence". $a = 0$ in eq.(1) will always give the usual "parallel coincidence". The limit in ϕ is obtained from

$$\phi < \arccos \left(\frac{f_c}{4f_o} \left(\sqrt{1 + (4f_o/f_c)^2} - 1 \right) \right) \quad (12)$$

For $f_o/f_c \ll 1$ we get

$$\phi < \arccos(2f_o/f_c) \quad (13)$$

For $f_o \ll f_c$, ϕ almost covers $0 - \pi/2$ according to (13), but if f_o is raised by a very narrow space we get a different solution.

For some years a new type of traffic noise windows [23] has been available with one pane + a double pane with 0.3 - 1 mm space. $f_c \approx 13000/$ (thickness in mm) for glass. With 4 mm pane and 0.5 mm space we get:

$$f_c/f_o \approx 2.7 \quad (14)$$

In the solution shown here there is a faster shift towards higher frequencies of f_{d1} with increasing ϕ_1 , compared to the limp wall solution. If we use (11) and compare with the classical limp wall f_{od} we get:

$$f_{od} = f_o / \cos \phi \quad (15)$$

$$\frac{f_d}{f_{od}} = \frac{f_c \cos \phi}{f_o \sqrt{2} \sin^2 \phi} \quad (16)$$

This fact together with the limited angle is probably also working on the elimination of an effective resonance in the TL-curve for diffuse sound fields.

3.8. The components of the multiple wall chain

When calculating the transmission we can use a network of the following type for spaces with $d < \lambda/6$:

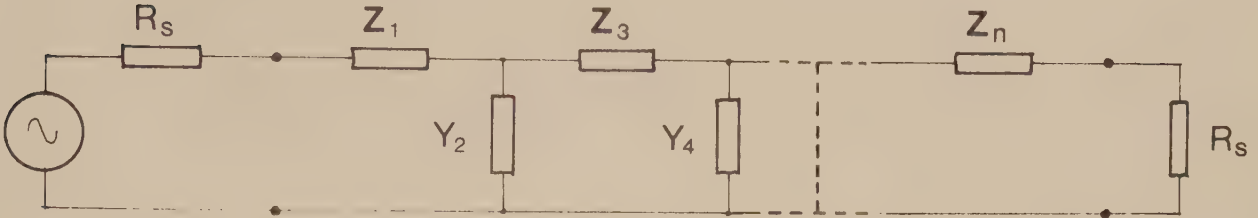


FIGURE 3.8.1. The multiple partition chain network.

For the infinite lossless case we get

$$R_s = \rho c / \cos \phi \quad (1)$$

(ϕ is the angle of incidence.)

In the lossless case the leaf impedances are

$$Z_n = j\omega M_n (1 - (k/\kappa_n)^4 \sin^4 \phi) \quad (2)$$

For empty spaces larger than 3 mm we get:

$$Y_n = \frac{jkd_n}{\rho c} \quad (3)$$

As usual we have

$$\kappa^4 = \omega^2 M/B \quad (4)$$

We insert the loss factor of the leaf η in B (solid friction of bending):

$$\bar{B} = B(1 + j\eta) \quad (5)$$

$$\bar{\kappa}^4 = \kappa^4 / (1 + j\eta) \quad (6)$$

$$\bar{Z}_n = j\omega M_n (1 - (k/\bar{\kappa}_n)^4 (1 + j\eta_n) \sin^4 \phi) \quad (7)$$

$$\operatorname{Re} \bar{Z}_n = \eta_n \omega M_n (k/\kappa_n)^4 \sin^4 \phi = B_n \eta_n (k \sin \phi)^4 \quad (8)$$

The parameter η often depends on the frequency. η is often very small, less than 10^{-2} . From par. 3.7 we obtain the space admittance for $d < 3$ mm or with filling ($r \neq 0$):

$$Y_n = \frac{jk d_n}{\rho c} \left(\frac{\gamma}{\mu} - \frac{\sin^2 \phi}{1 + r_n/(jk)} \right) \quad (9)$$

We wish to use the method for a limited area, e.g. a window. Then we shall take into account the limitation of R_s discussed in par. 2.4 and given in eq.(2.4.5):

$$R_s \approx \frac{\rho c}{\cos \phi + 4/(kl)} \quad (10)$$

Because of the boundaries we shall modify η by the following connection:

$$\eta' = \eta + \eta_{\text{bound}} \quad (11)$$

For Y_n there is the possibility of not being a developed spacewave. So it is possible that r' will virtually be larger than r for finite structures:

$$r' = r + r_{\text{limited}} \quad (12)$$

In this model the transmission decrease due to limited area has thus been included. In this chain filter method we have not taken into account the possibilities of excess transmission produced by boundaries. Such effects will be discussed in Part 4 for single partitions.

As η' and r' were not exactly known in the computations presented in the next paragraph I used tentative values. Therefore the results are not purely theoretical.

As (10) is a simplified form, also variations of R_s were allowed for by using an arbitrary constant G

$$R_s = \frac{\rho c}{\cos \phi + G/f} \quad (13)$$

and by varying G . There was a tendency to get better results for smaller G than could be expected from eq.(10). The probable explanation is the lack of boundary-induced transmission in this simple chain filter model. I shall in due course improve the chain model with boundary-scattered waves, as will be shown in Part 4 for a single partition.

3.9. A triple-pane "traffic noise window"

When trying to increase the TL for double windows the coincidence effect limits the optimal thickness of a pane.

According to [23] a good solution is to put an extra pane close to one of the panes of a double glazing. The resonance of this pair of panes will be high but has not appeared in TL-measurements. I therefore found it interesting to apply some analytical methods to the "traffic noise window".

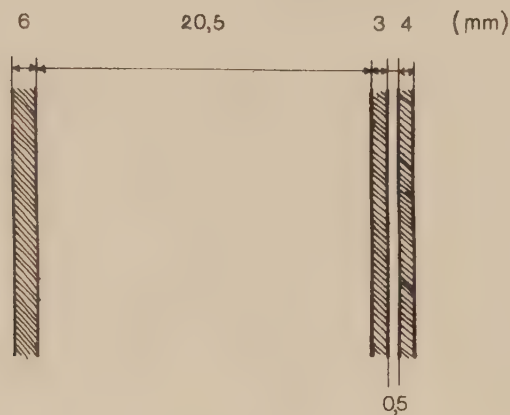


FIGURE 3.9.1. "Traffic noise window".

The components have been given previously as radiation resistance, pane and interspace admittance for small spaces. The network will be as follows:

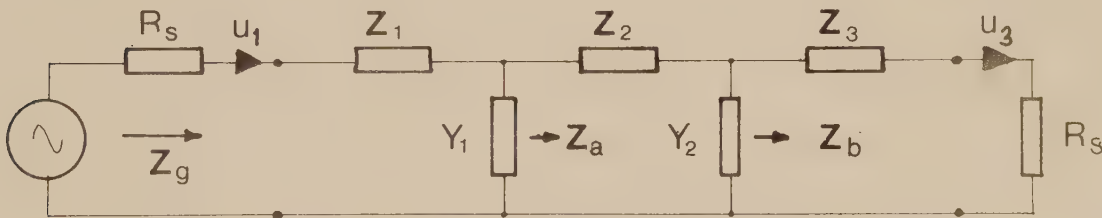


FIGURE 3.9.2. The window network.

$$Z_g = R_s + Z_1 + \frac{1}{Y_1 + \frac{1}{Z_2 + \frac{1}{Y_2 + \frac{1}{Z_3 + R_s}}}} \tag{1}$$

This can be splitted up and we get

$$Z_b = \frac{1}{Y_2 + 1/(Z_3 + R_s)} \quad (2)$$

$$Z_a = \frac{1}{Y_1 + 1/(Z_2 + Z_b)} \quad (3)$$

$$Z_g = Z_s + Z_1 + Z_a \quad (4)$$

$$u_1 = 2p/Z_g \quad (5)$$

$$u_3 = u_1(1 - Y_1 Z_a)(1 - Y_2 Z_b) \quad (6)$$

$$\tau' = \frac{|u_3|^2 R_s}{p^2/\rho c} \quad (7)$$

$$\tau_S = 2 \int \tau'(\phi) \sin \phi \, d\phi \quad (8)$$

During these computations τ' (see eq.(2.8.3)) turned out to be convenient as τ tended to $\rightarrow \infty$ for grazing sound.

In the computations I first tried the classical method, using pure limiting angle and unlimited grazing radiation resistance. However, this was not satisfactory. Especially the classical method seems not to be consistent, as different limit angles had to be used for 1, 2 and 3 panes.

In the following figures the results of some combinations are shown.

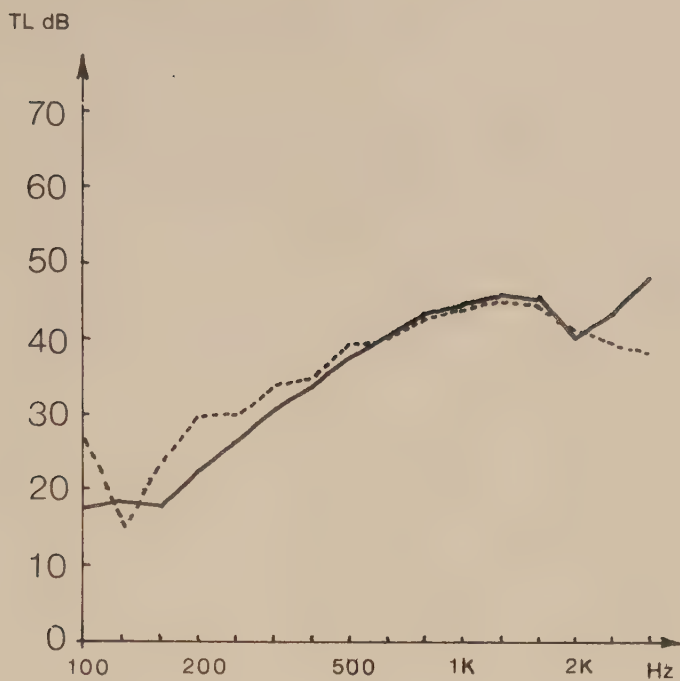


FIGURE 3.9.3. Measured and computed TL for the glazing: 6 - 20.5 - 3 - 0.5 - 4.

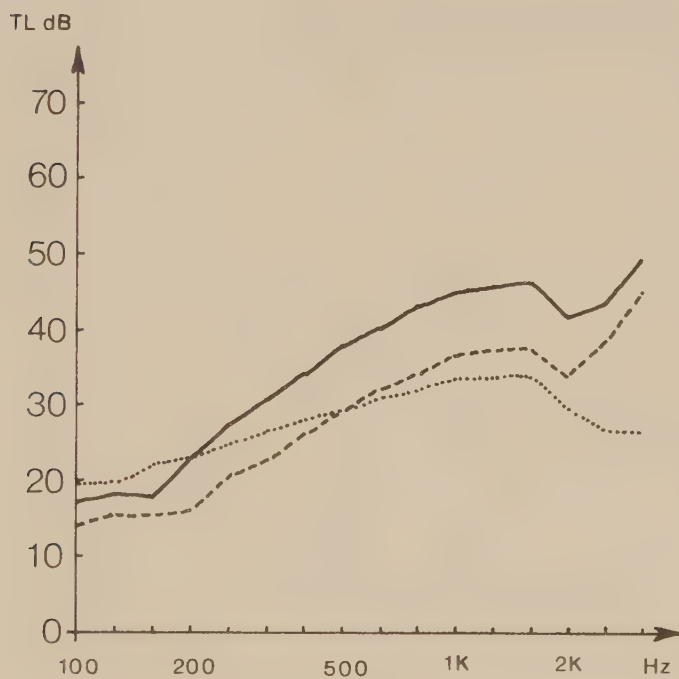


FIGURE 3.9.4. 1, 2 and 3 panes. Computed TL for single (6), double ---- (6 - 20.5 - 3) and triple — (6 - 20.5 - 3 - 0.5 - 4) glazing.

There is no sign of the resonance just above 1200 Hz in calculations or

3.10. Concluding remarks to Part 3

As we have seen, Part 3 is a connecting part between introduction and new developments. The methods introduced have been tested on acoustical subjects and I claim that the simplified wave modifications presented here can speed up the work on noise problems. In my opinion knowledge of some standard waves and how to modify them is essential and more useful than many spatial formulas and curves.

The narrow space waves have shown that a second hand picture like the reflexion method, is of minor **fitness** for use compared to the proper wave model. The latter can be modified for losses and other alterations, which is not the case for second hand descriptions.

I hope that the ideas demonstrated here can be adopted at different tasks, some of which have been shown already.

PART 4

POWER ENERGY (PE) AND MEAN SQUARE COUPLING (MSC) METHODS
APPLIED TO ROOM FIELDS AND REVERBERANT EDGE TRANSMISSION

4.1. Introduction

The classical Sabine theory is one of the first PE-models. An advanced form of PE-calculation is the statistical energy analysis (SEA) given by Lyon and Maidenek [29].

I intend to show further applications of power energy (PE) models, as the data reduction before calculation is very time-saving. Mean square coupling (MSC) means reduction of data, that is often redundant, by averaging after the calculation of the coupling.

Occasionally there is necessary information in MSC-calculations, that is not available in PE-results. Here it is shown that the room coupling with low acoustic generator impedance is such a case. If the coupling is not affecting the generator, it is shown that there is no meaning in the redundant information of MSC-calculations.

It is often possible to achieve a sufficient accuracy without complicated coupling methods. It will be shown that the first, natural step to correct the infinite case theory is to introduce a partition edge, a half-infinite plate and quarter-infinite air spaces. The edge radiation is shown to be an effect of "semi-coincidence" along the edge.

The edge-produced field is calculated by coupling factors and mode densities. A still simpler method is introduced, that involves "scattering" at the edge from the mass-controlled field. The word "scattering" is used, as the new wave is not reflected. The scattered field is also used in a PE-method, and the result is a simple correction to the masslaw.

Some discussions about equal rooms and improvements on the coupling theories are also delivered in this part.

4.2. Power-energy sources and flow sources in rooms

In the Sabine room the energy is built up by waves of all directions. In a point we then presume a stochastical intensity $\vec{J}$. For the expected values we get

$$E[\vec{J}] = 0 \quad (1)$$

$$E[|\vec{J}|] = I \quad (2)$$

I is a diffuse "magnitude intensity" that is practical to use. For the mean square pressure p^2 we get

$$I = \frac{\overline{p^2}}{\rho c} \quad (3)$$

With this definition "intensity level" will be equal to "sound pressure level".

The mean intensity towards a surface with a normal direction $\vec{n}$ into the surface will be

$$E[\vec{J} \cdot \vec{n} \text{ (if } > 0)] = \frac{I}{4} = I_0 \quad (4)$$

We define a diffuse α_s as the absorbed portion of I_0 . For the whole room the absorption area A is

$$A = \sum \alpha_{sn} \cdot S_n \quad (5)$$

For a total source power Π we get a power balance in the room

$$\Pi = I_0 A \quad (6)$$

$$I = \Pi / 4A \quad (7)$$

The energy density in a wave is $|\vec{J}|/c$, so in the room V we get the decay of the total energy

$$\frac{d}{dt} \left(\frac{I}{c} \cdot V \right) = - \frac{I}{4} A \quad (8)$$

$$I = I(t_0) \exp \left(- \frac{Ac}{4V} (t - t_0) \right) \quad (9)$$

By using I instead of $p^2/\rho c$ a shorter form is obtained. I also think that a well-defined intensity level is favourable to the fluctuating power $\Pi(t)$, as the "noise dose" in the room then will be

$$D = \int_{t_1}^{t_2} I(t) dt \quad (10)$$

For two rooms we define a transmission factor for I_{01} , τ , and for the partition area S we get

$$\Pi_2 = I_{01} \cdot \tau \cdot S = (I_1/4) \cdot \tau \cdot S \quad (11)$$

and with (7)

$$I_2 = I_1 \tau S/A \quad (12)$$

I would like to point out that intermediate formulas written in "levels" give long and unpractical expressions. Further discussion is given in [28].

In direct calculation of the sound field we have to use a flow source distribution $Q(\vec{r})$ in the room. We get the inhomogeneous wave equation:

$$(\Delta + k^2) p(\vec{r}) = jk\rho c Q(\vec{r}) \quad (13)$$

The eigenmodes $P_n(\vec{r})$ fulfil the boundary conditions and the wave equation:

$$(\Delta + k_n^2) P_n(\vec{r}) = 0 \quad (14)$$

We transform both sides of (13) on the modes:

$$p = \sum p_n P_n \quad (15)$$

$$Q = \sum q_n P_n \quad (16)$$

$$(-k_n^2 + k^2) p_n = jk\rho c \frac{\iiint_V P_n(\vec{r}) Q(\vec{r}) d\vec{r}}{\iiint_V P_n^2(\vec{r}) d\vec{r}} \quad (17)$$

The denominator normalizes the type of mode in the volume:

$$M_n = \iiint_V P_n^2(\vec{r}) d\vec{r} \quad (18)$$

$$q_n = \left(\iiint_V P_n(\vec{r}) Q(\vec{r}) d\vec{r} \right) / M_n \quad (19)$$

$$p_n = \frac{q_n}{-k_n^2 + k^2} jk\rho c \quad (20)$$

If $Q(\vec{r})$ is a point source at $\vec{r}_0$ we use Dirac $\delta(\vec{r} - \vec{r}_0)$. With the volume velocity $W \text{ m}^3/\text{s}$ we get the following connections for Q :

$$Q(\vec{r}) = W \cdot \delta(\vec{r} - \vec{r}_0) \quad (21)$$

and obtain

$$q_n = \frac{W P_n(\vec{r}_0)}{M_n} \quad (22)$$

The mean square pressure is $E[q^2] = \mu_{q2}$ or the estimate $\overline{q^2}$, often an integral obtained from RC- or ideal integration. For noise we have the spectral connection $S_2(\omega) = |H(\omega)|^2 S_1(\omega)$. Integrating over the band it gives the mean square $\mu_{x2} = \int |H(\omega)|^2 d\omega \cdot \mu_{x1}$, if $S_1(\omega)$ is filtered white noise.

In rooms or plates $H(\omega)$ is given by the modes and the integral of the sum of mode resonance integrals. First we shall study a room with a simple source.

4.3. Mean square pressure from mean square coupling (MSC) of a point volume velocity source

A room field is driven by a point source $w(t)$ with the time transform $W(\omega)$. With small losses η , k_n is replaced by $\bar{k}_n^2 = k_n^2(1 - j\eta)$ and we get the following total transfer function H from point source velocity to sound pressure in the room:

$$H(\omega) = \frac{p}{W} = jk\rho c \sum \frac{P_n(\vec{r}_o) P_n(\vec{r})}{M_n(k^2 - k_n^2(1 - j\eta))} \quad (1)$$

The mean square response $E[q^2]$ from the mean square volume velocity $E[w^2]$ is calculated from $E[|H|^2]$. To get the mean square response we integrate term by term over the resonance peaks. For $k \approx k_n$ we put $k = k_n + \Delta k$ and get

$$\begin{aligned} k^2 - k_n^2(1 - j\eta) &= (k + k_n)(k - k_n) + j\eta k_n^2 \approx \\ &\approx k_n^2 \left(j\eta + 2 \cdot \frac{\Delta k}{k_n} \right) \end{aligned} \quad (2)$$

For the resonant term P_{on} the peak value at resonance gives a peak transfer function H_{on} :

$$H_{on} = \frac{P_{on}(\vec{r})}{W} = \frac{\rho c}{k_n \eta} \cdot \frac{p_n(\vec{r}_o)}{M_n} \cdot P_n(\vec{r}) \quad (3)$$

Near the peak we obtain a resonance peak function:

$$H_n = H_{on}(\vec{r}) \frac{1}{1 - j \frac{2}{\eta} \cdot \frac{\Delta k}{k_n}} \quad (4)$$

We shall now integrate $|H_n|^2$ over the resonance. The integral is B_N . B_N can be used further on to find the total MS-response.

$$\begin{aligned}
 B_N &= \int_{-\Delta k_1}^{\Delta k_1} \left| \frac{H_n}{H_{no}} \right|^2 dk = \int_{-\Delta k_1}^{\Delta k_1} \frac{1}{1 + \left(\frac{2}{\eta} \frac{\Delta k}{k_n} \right)^2} d\Delta k = \\
 &= 2 \frac{\eta k_n}{2} \operatorname{arctg} \frac{2\Delta k_1}{\eta k_n}
 \end{aligned} \tag{5}$$

In spite of the expansion in Δk we let $\Delta k_1 \rightarrow \infty$ and get B_N . To distinguish k -bandwidth from peak bandwidth in ω or f we use B_{Nk} :

$$B_{Nk_n} = \frac{\eta k_n \pi}{2} \tag{6}$$

A square peak with the width B_N has also an integral B_N . B_N is referred to as the noise bandwidth, as it directly gives the mean square response to white noise, if the peak response is known. We get the total response of white noise in a band k_B with N modes from the sum of integrals over the mode resonance peaks:

$$\mu_{q^2}(\vec{r}) = \sum_1^N |H_{on}|^2 (B_{Nk_n}/k_B) \cdot \mu_{w^2} \tag{7}$$

If the resonances have equal bandwidths and a mean distance $k_N = k_B/N$, eq.(7) is simplified. The mean peak response is:

$$\begin{aligned}
 |H_o|^2 &= \sum_1^N |H_{on}|^2 / N \\
 \mu_{q_o^2} &= |H_o|^2 \mu_{w^2}
 \end{aligned} \tag{8}$$

Hence the mean square response in the room is:

$$\begin{aligned}
 \mu_{q^2} &= E \left[\mu_{q_o^2} \frac{B_{Nk}}{k_N} \right] = E \left[|H_o|^2 \frac{B_{Nk}}{k_N} \mu_{w^2} \right] = \\
 &= \mu_{w^2} \left(\frac{\rho c}{k\eta} \right)^2 \cdot \frac{B_{Nk}}{k_N} \cdot E \left[P_n^2(\vec{r}_o) \cdot P_n^2(\vec{r}) / M_n^2 \right]
 \end{aligned} \tag{9}$$

The resonance peak can be replaced by a square peak with the width B_N . If the mean peak distance k_N is equal to B_N the mean response of all modes is equal to the peak response of one mode.

The mode distance in a room is found from the usual mode lattice. All modes below k are restricted by the following inequality:

$$k_{nx}^2 + k_{ny}^2 + k_{nz}^2 \leq k^2 \quad (10)$$

This is an eighth of a sphere. The k_n -distances in an abd m^3 room are π/a , π/b , π/d . We get the number of modes from the volume, plane surfaces and edges of the eighth sphere:

$$N_3 = \frac{1}{6\pi^2} k^3 abd = \frac{k^3 V}{6\pi^2} \cdot (\text{full modes, 3-modes}) \quad (11)$$

$$N_2 = \frac{k^2}{4\pi} (ab + bd + da) = \frac{k^2 S}{4\pi} \quad (12)$$

$$N_1 = \frac{k}{\pi} (a + b + d) = \frac{k l}{\pi} \quad (13)$$

The corresponding mode densities on the k -axis are

$$\frac{dN_3}{dk} = \frac{k^2 V}{2\pi^2} \quad (14)$$

$$\frac{dN_2}{dk} = \frac{kS}{2\pi} \quad (15)$$

$$\frac{dN_1}{dk} = \frac{l}{\pi} \quad (16)$$

The mode distance is

$$k_N = \frac{1}{dN/dk} \quad (17)$$

$$k_{N3} = \frac{2\pi^2}{k^2 V} \quad (18)$$

$$k_{N2} = \frac{2\pi}{kS} \quad (19)$$

$$k_{N1} = \frac{\pi}{l} \quad (20)$$

We introduce a relative mode distance Δ_r :

$$\Delta_r = \frac{k_n}{B_{Nk}} = \frac{k_B}{NB_{Nk}} \quad (21)$$

If $\Delta_r < 1$ we have "mode overlap". Now we can write μ_{q2} as:

$$\begin{aligned} \mu_{q2} &= \mu_{q0}^2 / \Delta_r \\ \mu_{q0}^2 &= \mu_w^2 \left(\frac{\rho c}{k\eta} \right)^2 E \left[P_n^2(\vec{r}_0) P_n^2(\vec{r}) / M_n^2 \right] \end{aligned} \quad (22)$$

The space average of (22) is obtained from the fact that $E[M_n] = E[P_n^2] \cdot V$:

$$E \left[\left(\frac{P_n^2(\vec{r}_0) P_n^2(\vec{r})}{M_n^2} \right) \right] = \frac{1}{V^2} \quad (23)$$

Hence we finally get the **average** sound pressure in a room:

$$\mu_{q2} = \frac{\mu_{q0}^2}{\Delta_r} = \left(\frac{\rho c}{k\eta V} \right)^2 \cdot \frac{1}{\Delta_r} \mu_w^2 \quad (24)$$

At higher frequencies 3-modes dominate and with eqs.(6) and (14) we get the final result:

$$\mu_{q2} = \mu_{q0}^2 \cdot \frac{\pi k \eta}{2} \cdot \frac{k^2 V}{2\pi^2} = \frac{(\rho c)^2 k}{V \cdot 4\pi \eta} \mu_w^2 \quad (25)$$

From the details of the response we have reduced data to obtain a simple expression. In the next paragraph the reduction of information is done before the start of calculations. This is referred to as a "PE-method" and will be compared with the mean square of the coupling (MSC) of this paragraph.

results, as the volume velocity must be unaffected by the produced pressure.

With finite Z_G the flow depends on the pressure produced by Z_S . We shall therefore study a small ($r \ll 1/k$) sound source being a non-perfect volume velocity source with the finite generator impedance Z_G . The "weakness" shunts the flow and makes the flow into Z_S less, especially if $Z_S > Z_G$. We can also say that the "force" is not powerful enough to keep up the flow velocity with a present pressure.

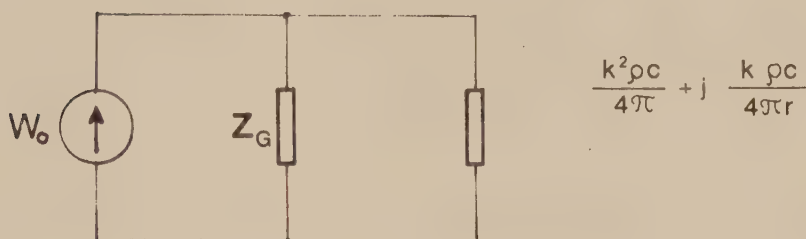


FIGURE 4.4.1. The PE-radiation network.

We start with the simple PE-model, where the pressure is assumed to be the same as in free space at the source:

$$\text{PE: } \frac{\mu_{\Pi Z_G}}{\mu_{\Pi \infty}} = \left| \frac{Z_G}{Z_G + Z_S} \right|^2 = \frac{1}{|1 + Z_S/Z_G|^2} \quad (5)$$

In the MSC-model we use the real pressure at resonance. From eq.(4.3.24) we get a new load resistance R_M at resonance, which can be larger than R_S , as the pressure at resonance is larger than the free space pressure.

$$R_M = \frac{\rho c}{k\eta V} \quad (6)$$

Also in this case we can use a network to describe the radiation of the flow:

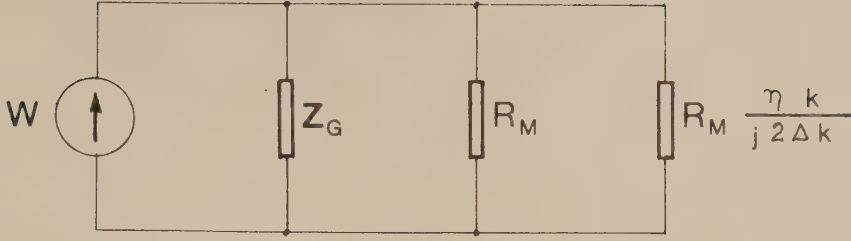


FIGURE 4.4.2. The MSC-radiation network.

Z_G will lower the peak but also make it wider, so we get a modified MSC-power:

$$\text{MSC: } \frac{\mu_{\Pi, Z_G}}{\mu_{\Pi, \infty}} = \left| \frac{1}{1 + \frac{R_M}{Z_G}} \right|^2 \left| 1 + \frac{R_M}{Z_G} \right|^1 = \left| 1 + \frac{R_M}{Z_G} \right|^{-1} \quad (7)$$

The results can be compared with the SEA-method in [29] and [9].

We compare the PE- and the MSC-results:

$$\frac{\text{MSC: } \mu_{\Pi}}{\text{PE: } \mu_{\Pi}} = \frac{\left| 1 + \frac{k \rho c}{4 \pi} \left(k + \frac{j}{r} \right) / Z_G \right|^2}{\left| 1 + \rho c / (k \eta V Z_G) \right|} \quad (8)$$

The real and imaginary part of Z_s are:

$$R_s = k^2 \rho c / 4 \pi \quad (9)$$

$$x_s = R_s \cdot j / (k r)$$

With the relative mode distance Δ_r we get

$$\Delta_r = \frac{4 \pi}{k^3 \eta V} \quad (10)$$

$$R_M = R_s \cdot \Delta_r \quad (11)$$

$$\frac{\text{MSC: } \mu_{II}}{\text{PE: } \mu_{II}} = \frac{\left| 1 + \frac{R_s}{Z_G} \left(1 + \frac{j}{kr} \right) \right|^2}{\left| 1 + \frac{R_s}{Z_G} \Delta_r \right|} \quad (12)$$

As

$$\frac{1}{k\eta} \approx 25 T_{60} \quad (13)$$

Δ_r can be expressed in T_{60} :

$$\Delta_r \approx \frac{310 T_{60}}{k^2 V} \quad (14)$$

The reverberation room of our department (Building Acoustics, LTH) has 200 m³ volume and Δ_r -values of 5 - 10 can be obtained. Bodlund [30] has reported effects on mode response and T_{60} . He used a loudspeaker cabinet with a base reflex opening that has rather low R_s at (reflex) resonance.

I have made some more experiments concerning the influence of Z_g , as power measurements have an increased interest in practice. The power has been measured in the anechoic chamber and in the reverberant room.

A loudspeaker membrane has a generator resistance R_G of the following type:

$$R_G = \frac{1}{S_{\text{member}}^2} \frac{(Bl)^2}{R_{\text{coil}}} \quad (15)$$

(Bl) is the field $\times$ length of coil wire. R_G is normally very large compared to both R_M and R_s . We had to use a quarter wavelength transformer to obtain the wanted generator with small Z_G for the experiments:

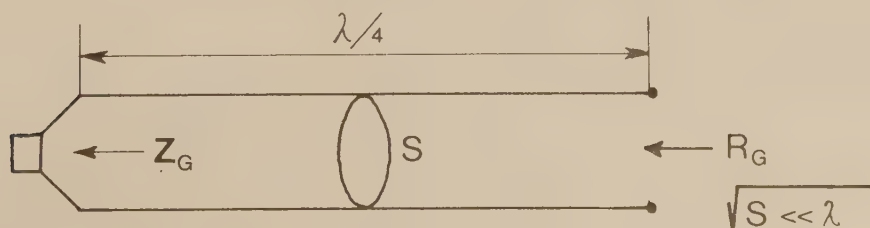


FIGURE 4.4.3. The low Z_G transformer.

$$Z_G = \frac{(\rho c / S')^2}{Z'_G} \quad (16)$$

The results were as follows. With noise of 10 Hz bandwidth the "transformer" increased the power from a Goodman maxim loudspeaker by 11 dB in the anechoic chamber and by 8 dB in a reverberant room (100 m^3 and with $\Delta_r = 6$).

In a special test the strong mode was driven from a corner at resonance with a pure tone. The peak power increase of the transformer was 17 dB in the anechoic chamber, but only 11 dB when the strong mode was driven. The quotient $R_M/R_S = \Delta_r$ was equal to 10 calculated from the reverberation time.

For some sources, discrepancies between power calculated from sound pressure in an anechoic chamber and a reverberant room, can perhaps be explained by the related theory and experiment. However, I shall not judge the importance of this result.

Often the pressure on a wall is not affected considerably by the vibration and we have a pressure generator. Also in this case MSC- and PE-models give the same result.

For TL-calculations, the PE-models are therefore fit for use both for the step room - wall and wall - room. Also the used MSC-methods usually assume pure pressure and velocity generators respectively.

4.5. The power energy network for a room

We shall now study the time dependence and modulation transfer function for noise in a room. We shall use an equivalent network, that has an analogy different from earlier used networks. We get the following circuit for power and energy in a room:

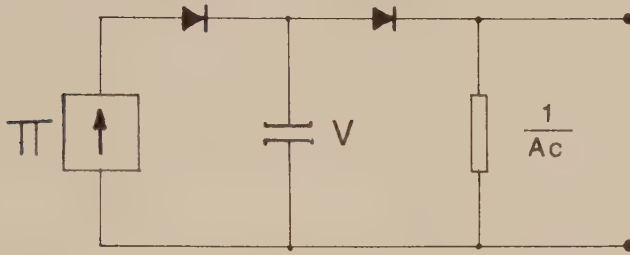


FIGURE 4.5.1. Power energy analogous network.

The rectifiers have been included to show the power current into the room and the absorbing area only. We get a new PE-transfer function G for the modulation of the power Π :

$$\Pi_t = \Pi_o (1 + M \sin(\Omega t)) \quad (1)$$

$$E(\Omega) = \Pi_o \frac{4}{Ac} + \Pi_o M \frac{1}{\frac{Ac}{4} + j\Omega V} \quad (2)$$

For very low modulation frequency E is modulated just as Π .

The modulation transfer function G_Ω has the following connection, which shows a decreasing modulation and a phase difference for increasing Ω :

$$G = \frac{1}{\frac{Ac}{4} + j\Omega V} = \frac{4}{Ac} \frac{1}{1 + j\Omega/\Omega_g} \quad (3)$$

G is simply the transform of the energy impulse response, the usual reverberation curve.

We can observe that $\Omega = \Omega_g$ gives $E_{\max} \sqrt{2}$, so the "3 dB drop" of the net-

work is only a 1.5 dB drop with the usual definition of dB (dB: $10 \log E$ but $20 \log (\text{voltage})$). The "power impulse response" is the inverse transform and $\Omega = Ac/4V$ gives a pole, so we get:

$$E(t) = E(0) \exp \left(-\frac{Ac}{4V} t \right) = E(0) \exp(-t/\tau_{\Omega}) \quad (4)$$

τ_{Ω} is the energy time constant:

$$\tau_{\Omega} = \frac{4V}{Ac} = \frac{1}{\Omega_g} \quad (5)$$

Note that the normal physical τ_e (time constant) is connected with the amplitude and defined as

$$\tau_e = \frac{1}{\omega\eta/2} \quad (6)$$

The reverberation time, T_{60} , can be expressed in τ_{Ω} or τ_e :

$$e^{-T_{60}/\tau_{\Omega}} = 10^{-6} ; \quad e^{-T_{60}/\tau_e} = 10^{-3} \quad (7)$$

$$T_{60} = 6 \cdot \ln 10 \tau_{\Omega} = 13.8 \tau_{\Omega} \quad (8)$$

$$T_{60} = 3 \cdot \ln 10 \tau_e = 6.9 \tau_e \quad (9)$$

It is of course an old idea to regard the room as a low pass filter [32]. A report [33] has also been communicated, showing one treatment of a model. In 1972 I reported [34] some measured results of a room as low pass filter according to the described model. We used the following circuit to get true sinusoidal modulation of the power:

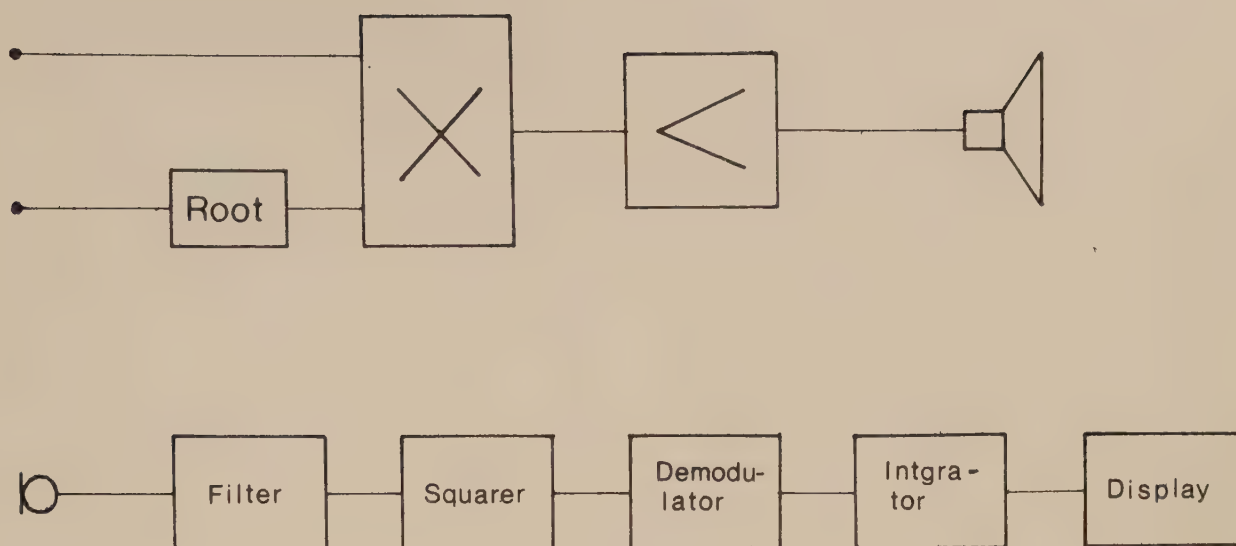


FIGURE 4.5.2. The power modulation set-up.

A simple low-pass integration was used. To get a better result one has to use band pass filter at Ω or signal recovery of a correlator [33]. The last-mentioned method will be adopted in our further experiments.

Thus we had difficulties in the lower bands but got the following approximate G -curves:

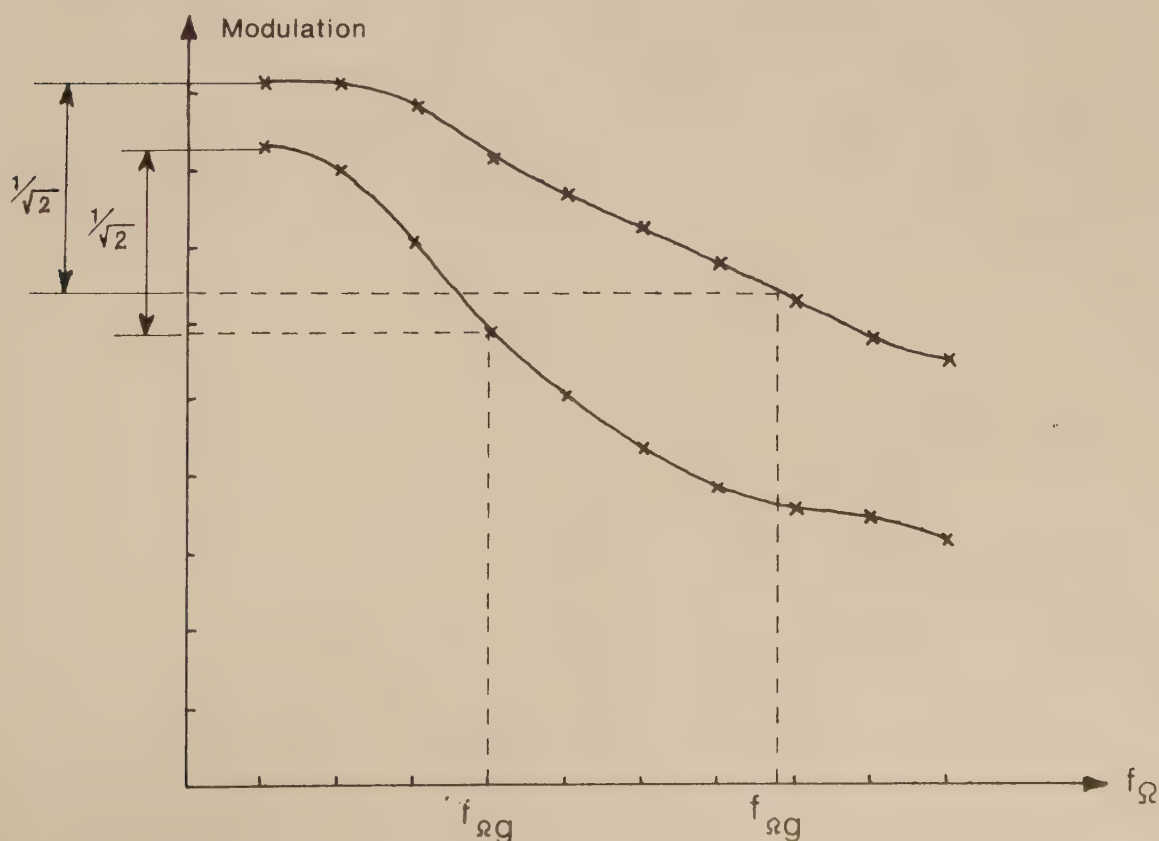


FIGURE 4.5.3. Energy modulation.

From $f_{g\Omega}$ we obtain $\tau_{\Omega} = 1/(2\pi f_{g\Omega})$ and $T_{60,\Omega}$, which can be compared with normal reverberation, T_{60} , from decay curves:

Octave band		250	500	1k	2k	4k	Hz
Empty room	$T_{60,\Omega}$	4.7	5.0	5.4	4.2	2.8	s
	T_{60}	8.0	8.0	7.0	5.0	3.0	s
Absorbent in the room	$T_{60,\Omega}$	1.7	2.1	2.2	1.9	1.6	s
	T_{60}	3.0	2.3	2.1	1.9	1.7	s

Table 4.5.1. Normal and modulation reverberation time.

The systematic differences at lower frequencies, especially for an empty room, could possibly be eliminated by proper signal recovery.

G has also been used in connection with articulation tests in rooms as a new measure of the useful reflexions, [35] and [36]. If we have $\Omega = \Omega_g$ computed from T_{60} , but get more than $1/\sqrt{2}$ of the modulation, there are probably special reflexions not taken into account in the PE-model. Therefore $G(\Omega_g)$ can be used as a measure of the presence of "useful" reflexions. A favourable property is that the value of G goes towards $1/\sqrt{2}$ for rooms where early reflexions have little influence. With the pulse method referred to as "Deutlichkeit" [36] we lack a similar reference value.

4.6. The pulse response of one and two rooms

We shall now adopt the model in the preceeding paragraphs to a single partition $S \text{ m}^2$ with the losses $\omega\eta$. We introduce K_1 and K_2 for the coupling room - wall and wall - room.

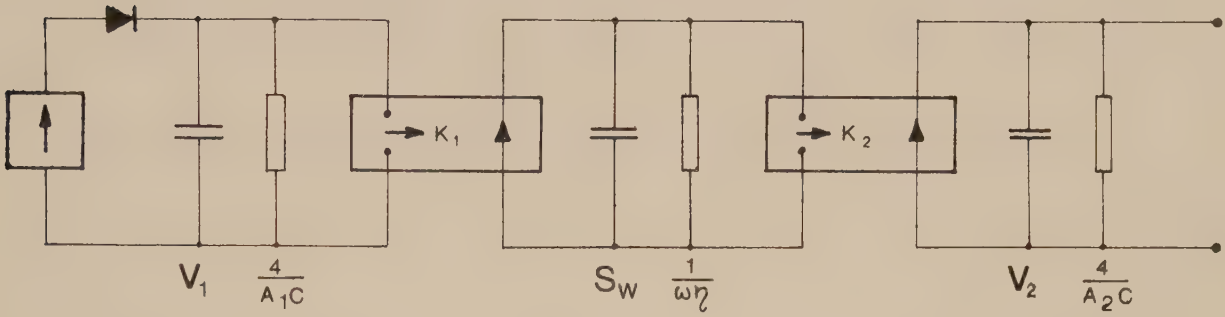


FIGURE 4.6.1. An energy power network for two rooms and partition.

$$E_1(\Omega) = \Pi \cdot G_1 \tag{1}$$

$$E_2(\Omega_2) = \Pi \cdot G_1 \cdot K_1 \cdot G_w \cdot K_2 \cdot G_2 \tag{2}$$

In [37] an energy impulse or shot method for TL-measurements is reported to give the same result as normal TL-measurements with steady state noise.

The theoretical deduction is simple if the PE-model above is used. The steady state noise is considered a noise step and the shot a noise pulse. The integration of a pulse is a step. The shot method includes integration of the response. If the total G to room 2 is G'_2 we get the following:

$$\text{Noise step: } \frac{1}{j\Omega} \cdot G_1 \quad (\text{room 1}) \tag{3}$$

$$\frac{1}{j\Omega} \cdot G'_2 \quad (\text{room 2}) \tag{4}$$

$$\text{Integrated pulse: } G_1 \cdot \frac{1}{j\omega} \quad (\text{room 1}) \tag{5}$$

$$G'_2 \cdot \frac{1}{j\omega} \quad (\text{room 2}) \tag{6}$$

Of course this gives the same τ and TL with both methods, but it is doubtful whether it is a proof. In the deduction given in [37] the usual transfer functions of the filters $H(\omega)$ but also $G(\Omega)$ are used. This is an unpermitted mixture of two methods. (In [37] convolution is formally used, but this makes no difference.)

Also the reverberation pulse method can be treated similarly. This is not a proof but a model for reasoning.

The reverberation is the response of a negative power step:

$$\Pi \left(1 - \frac{1}{j\Omega} \right) \quad (7)$$

The step answer is the integral of a step and the impulse answer $h_{\Omega}(t)$. We can get the reverberation curve $x_{\Omega}(t)$ from the response $x_{p\Omega}$ of an energy pulse:

$$\begin{aligned} x_{\Omega}(t) &= \left(1 - \int_0^t h_{\Omega}(t) dt \right) (\text{pulse energy}) = \\ &= \int_0^{\infty} x_{p\Omega}(t) dt - \int_0^t x_{p\Omega}(t') dt' \end{aligned} \quad (8)$$

This can also be written in the following form:

$$x_{\Omega}(t) = \int_t^{\infty} x_{p\Omega}(t') dt' \quad (9)$$

To get (8) out of measurements we have to measure twice and for (9) a recording is made, which is played backwards. Schroeder's pulse method [38] and Kuttruff's method [39] are similar to eqs.(9) and (8). The energy model used here is however no proof for the use of the squared impulse response but a hint about the possibility of it being used for simplified educational purposes.

4.7. The coupling of a pressure to a homogeneous wall

In the TL-calculations we shall first calculate the transfer function of the different steps. We shall try to make the steps as small as possible to get simple transfer functions.

When calculating $H(\omega)$, as usual we can look upon the transforms P and u as pure tone signals. Then we use $|H(\omega)|^2$ to get the response of the noise.

We shall start with the first step in an MSC-TL-calculation. We assume room 1 to be a pressure generator. The pressure field $p(x, y)$ gives the following eigenfunction solution ($\kappa^4 = \omega^2 M/B$, B = plate bending stiffness) for the wall loaded by the pressure. The wall has homogeneous boundary conditions.

$$\Delta(\Delta u) B - \omega^2 u M = j\omega p \quad (1)$$

The eigenmodes U_n are U_n -solutions of the homogeneous wave equation:

$$\left. \begin{aligned} \kappa^4 &= \omega^2 M/B \\ \Delta(\Delta u) &= \kappa^4 u \end{aligned} \right\} \quad (2)$$

For $\Delta(\Delta U_n)$ we therefore get:

$$\Delta(\Delta U_n) = \kappa_n^4 U_n \quad (3)$$

We now transform both sides of (1) on the U_n -terms and obtain:

$$u = \sum \sum u_n U_n \quad (4)$$

$$p = \sum \sum p_{un} U_n \quad (5)$$

(4) is the eigenmode series of u and (5) is the Fourier series of p on the eigenmodes of u . Thus we get:

$$p_{un} = \frac{\iint p U_n dS}{\iint U_n^2 dS} \quad (6)$$

$$u_n = \frac{p_{un} j\omega/B}{\frac{l}{\kappa_n} + \frac{l}{\kappa}}$$

$$M_{un} = \iint U_n^2 dS \quad (7)$$

n is an array of two numbers.

If the dominating part of eq.(4) comes from $\kappa_n \ll \kappa$ we get

$$u \approx -\Sigma \frac{p_{un} \cdot j\omega}{M\omega} = \frac{P}{jM\omega} \quad (8)$$

As discussed in Part 1 u'_n is the Fourier series in U_n of the mass-controlled velocity:

$$u = \frac{p}{j\omega M} \quad (9)$$

On the other hand, if we obtain $\kappa^4 \ll \kappa_n^4$ ($\omega \rightarrow 0$) we get the series for static pressure load. We have to use the displacement ζ :

$$\zeta_n = \frac{u_n''}{j\omega} = \frac{1}{B} \cdot \frac{p_{un}}{\frac{l}{\kappa_n} M_{un}} \quad (10)$$

$$\zeta = \Sigma \zeta_n U_n \quad (11)$$

$u_n \approx u_n''$ is referred to as the stiffness-controlled frequency range.

The U_n -terms are different for different boundaries. A review of the modes is given in [40] for simply supported (SS), clamped (C) and free (F) boundaries in different combinations. For supports with finite impedances ($\neq 0, \neq \infty$) U_n -forms are dealt with in [41] and [10]. Some cases can give u_n of known mathematical functions, e.g. SS on all edges.

For a normally incident wave p is constant over the plate ab . We get the

following three series for the simply supported case with constant pressure:

$$u_{mn} = \left(\frac{l}{\pi} \right)^2 \frac{p}{j\omega M} \cdot \frac{1}{mn(1 - (\kappa_{mn}/\kappa)^4)} \quad (12)$$

$$u'_{mn} = \left(\frac{l}{\pi} \right)^2 \frac{p}{j\omega M} \cdot \frac{1}{mn} \quad (13)$$

$$\xi = \frac{u_m}{j\omega} = \left(\frac{l}{\pi} \right)^2 \frac{p}{B} \cdot \frac{1}{\kappa_{mn}^4 \cdot mn} \quad (14)$$

The resonance wavenumbers κ_{mn} are:

$$\kappa_{mn}^2 = \left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \quad (15)$$

The resonance frequencies are:

$$f_{mn} = \frac{c\kappa_{mn}}{2\pi} \quad (16)$$

For increasing mn with $\kappa_{mn}^2 \approx mn$ we get the following behaviour of the series:

$$u'_{mn} \sim \frac{1}{mn} \quad (17)$$

$$\xi_{mn} \sim \frac{1}{(mn)^5} \quad (18)$$

As we know, sharp forms give strong terms for high numbers. The masslaw gives "impossible" boundary forms with steps, so (17) converges slowly. (12) is "softer" than (13) because of the resonance. We get the following picture of the results for the three series:

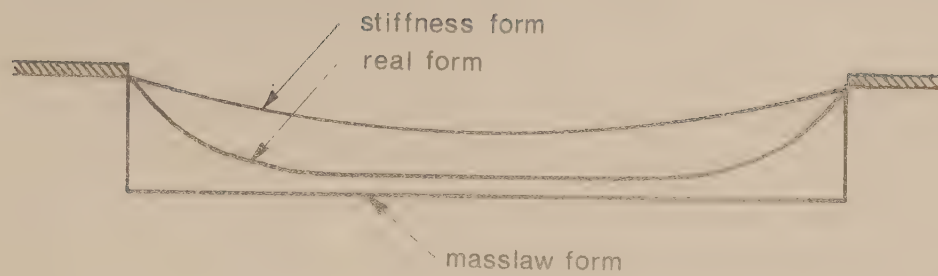


FIGURE 4.7.1. Real form, massform and stiffness form.

The result for static load is also well-known in applied mechanics [42].

4.8. The forced massfield and the resonant field

As the radiation and coupling is quite different for terms with $\kappa_n < k$ and with $\kappa_n > k$, it is convenient to divide eqs.(4.7.5) and (4.7.6) into terms with good coupling to the source field and terms with resonance in the source frequency band. Coupling and resonance cannot both occur for one term below coincidence ($k < \kappa$). If f_c is not too low the first terms u_n are equal to u'_n , that is

$$u' = \sum_1^{\infty} \sum_1^{\infty} u'_n U_n \approx \sum \sum u_n U_n \quad (\text{only non-resonant terms}) \quad (1)$$

The u'_n -terms give about the same result as the non-resonant u_n -terms. We refer to these terms in u_n as the masslaw part of u and put the result of the terms together as follows from the masslaw:

$$u_M = \frac{p}{j\omega M} \quad (2)$$

As u_M follows p according to (2) it has the same wave number as p , which is obtained from the incident sound field.

$$\kappa_M = \kappa_1 = k \sin \phi \quad (3)$$

u_M therefore radiates in the direction ϕ and with real impedance

$$Z_s = \rho c / \cos \phi \quad (4)$$

Under certain circumstances the resonant terms can add considerably to u_M . We insert a loss factor and obtain:

$$\bar{B} = B(1 + j\eta) \quad (5)$$

$$\bar{\kappa}_n^4 = \kappa_n^4(1 - j\eta) \quad (6)$$

Near a resonance we can use the following substitution:

$$\kappa^4 = (\kappa_n + \Delta\kappa)^4 \approx \kappa_n^3(\kappa_n + 4\Delta\kappa) \quad (7)$$

$$u_n = \frac{p_{un}}{\omega M(\eta + 4j(\Delta\kappa/\kappa))} \quad (8)$$

The peak transfer function from pressure to velocity is:

$$H_{on} = \frac{u_n}{p_{un}} = \frac{1}{\omega M\eta} \quad (9)$$

The peak integral gives a noise bandwidth (cf. par. 4.3):

$$B_{N\kappa} = \kappa \frac{\eta\pi}{4} \quad (10)$$

For comparison we shall also look at the corresponding f-bandwidth:

$$f = \sqrt{B/M} \cdot \kappa^2 \quad (11)$$

$$df = \sqrt{B/M} \cdot 2\kappa d\kappa \quad (12)$$

$$\frac{df}{f} = 2 \frac{d\kappa}{\kappa} \quad (13)$$

Hence the bandwidth in f is

$$B_{Nf} = f\eta \frac{\pi}{2} \quad (14)$$

In f or ω the bandwidth is the same for all types of waves ($\eta\pi/2$). The bandwidth $\kappa(\eta\pi/4)$ is a result of the group velocity being twice the phase velocity for bending waves.

The mode densities in κ have the same form as for k-values in the room. With new definitions of S, total area, and l, total edge length, we get (cf. par. 4.3):

$$\frac{dN_2}{d\kappa} = \frac{\kappa ab}{2\pi} = \frac{\kappa S}{2\pi} \quad (15)$$

$$\frac{dN_1}{d\kappa} = \frac{a + b}{\pi} = \frac{l}{\pi} \quad (16)$$

$$\kappa_{N2} = \frac{2\pi}{\kappa S} \quad (17)$$

$$\kappa_{N1} = \frac{\pi}{1} \quad (18)$$

The relative mode distances are

$$\Delta_{r2} = \frac{2\pi}{\kappa S} \cdot \frac{4}{\pi \kappa \eta} = \frac{8}{\kappa^2 S \eta} \quad (19)$$

$$\Delta_{r1} = \frac{\pi}{1} \cdot \frac{4}{\pi \kappa \eta} = \frac{4}{1 \kappa \eta} \quad (20)$$

The mean square resonant field is the squared peak response $|H_0|^2$, divided by the relative mode distance and multiplied by a coupling factor C:

$$\mu_{v2} = |H_0(\omega)|^2 C \mu_{q2} = \frac{C}{(\omega M \eta)^2 \Delta_r} \mu_{q2} \quad (21)$$

Compared to the massfield we get

$$Y = \frac{\mu_{v2res}}{\mu_{v2mass}} = \frac{C}{\eta^2 \Delta_r} \quad (22)$$

In [5] a calculation is performed for a beam leading to eq.(22) with Δ_r according to eq.(20) It is then used for a wall, which means that only 1-modes are considered. This is a reasonable assumption as we shall see later.

p_{un} are the Fourier terms of p on wall eigenmodes. p is the field of room modes P_m in the driving room:

$$p = \sum p_m P_m \quad (23)$$

The coupling C_{mn} of the room modes P_m to the wall modes U_n is:

$$C_{mn} = E \left[\frac{\iint P_m U_n dS}{M_{un}} \right] \quad (24)$$

For resonant wall modes far below coincidence $n \gg m$, so C is small and often depending on the form of P_m and U_n along the edges. Maidenek [43] describes the corner and edge coupling modes. In practice the edge coupling is often far more important than the corner coupling. Hence the latter is omitted and a half-infinite plate edge coupling is used in the next paragraph.

4.9. The coupling along an edge for $f < f_c$

As mentioned in Part 2 a suitable transform over the wall is of P -type, fitting the receiving room field. Then the coupling is simply the transform of U_n on P_q , so we get C_{nq} besides C_{mn} according to eq.(4.8.24). The coupling over an area is very weak if (k_x, k_y) of P_m or P_q are different from (κ_x, κ_y) of U_n . This is the case for resonant terms below the critical frequency, as $k_1 = k \sin \phi$ means that $k_1 < k < \kappa$. Then the integrals giving C cancel out over an area but show some uncanceled remainders at edges and corners. In [9] Maidenek showed transforms and coupling and in [43] he has given a recent survey of classification. Also [44] and [45] include many theoretical and experimental facts about corner and edge transmission.

Here the aim is to show a simple model for corrections to the infinite wall formulas. We shall therefore omit the corner coupling and only use edge coupling. If we have "semi-coincidence" between the air and plate mode along the edge, it means that the wavenumbers κ and k are equal along the edge ($\kappa_e = k_e$). To get the coupling we shall use the wavenumber κ_p (perpendicular to the edge), which we obtain from the bending wave equation and the boundary conditions:

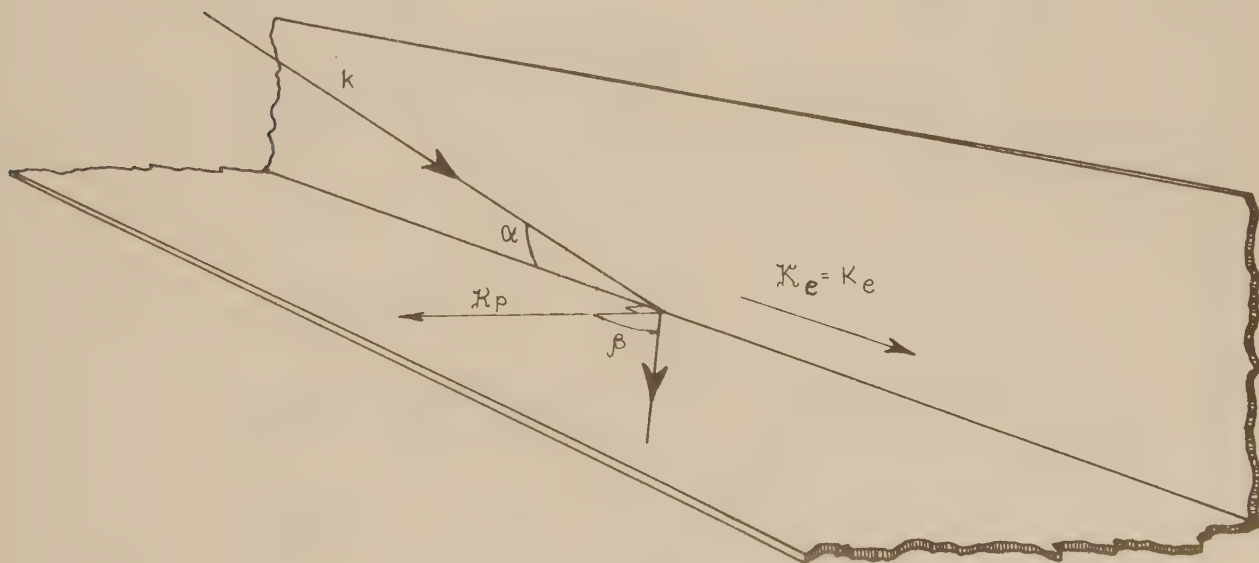


FIGURE 4.9.1. The edge coupling.

α is the angle between incident sound and the edge. β is the angle between a scattered bending wave and edge normal. From the plate equation we get ($\kappa_e = k_e$):

$$(\kappa_p^2 + k_e^2)^2 = \kappa^4 \quad (1)$$

$$\kappa_p^2 = \pm \kappa^2 - k_e^2 \quad (2)$$

Transmitting field, κ -numbers:

$$\kappa_p = \pm \kappa \sqrt{1 + (k_e/\kappa)^2} \quad (3)$$

Nearfield, κ -numbers:

$$\kappa_{pj} = \pm j\kappa \sqrt{1 + (k_e/\kappa)^2} \quad (4)$$

We can rewrite the obtained expressions by using the connection:

$$\kappa = k \sqrt{f/f_c} \quad (5)$$

Along the edge we have:

$$k_e = k \cos \alpha \quad (6)$$

(3) is rewritten:

$$\kappa_p = \pm \kappa \sqrt{1 + (f/f_c) \cos^2 \alpha} \quad (7)$$

(4) is rewritten:

$$\kappa_{pj} = \pm j\kappa \sqrt{1 + (f/f_c) \cos^2 \alpha} \quad (8)$$

κ_{pj} is the nearfield. For κ_p and κ_e we obtain the following formulas:

$$\kappa_p = \kappa \cos \beta \quad (9)$$

$$\kappa_e = \kappa \sin \beta \quad (10)$$

$$\kappa \sin \beta = k \cos \alpha \quad (11)$$

$$\beta = \arcsin\left(\frac{k}{\kappa} \cos \alpha\right) \quad (12)$$

$$\beta = \arcsin\left(\sqrt{f/f_c} \cos \alpha\right) \quad (13)$$

Far below the critical frequency ($f \ll f_c$) the plate wave is almost normal to the edge. β increases when $f \rightarrow f_c$. Eqs.(7) and (8) also give $\kappa_p \approx \kappa_{pj} \approx \kappa$ for $f \ll f_c$.

At an edge we must have a standing wave that can fulfil the boundary conditions. We shall use a simple method with a free incident bending wave ($0.5 \exp(j\kappa_p x)$). This is reflected with the factors A and B:

$$\frac{1}{2} \left(e^{j\kappa_p x} + A e^{-j\kappa_p x} + B e^{-\kappa_{pj} x} \right) = u_e \quad (14)$$

The different boundary conditions give different types of u_e . Two of the following functions, u , $\partial u / \partial x$, $\partial^2 u / \partial x^2$ and $\partial^3 u / \partial x^3$ must be zero in each boundary type. The result of this is:

$$\text{SS, simply supported: } u_e = \frac{1}{2} \left(e^{j\kappa_p x} - e^{-j\kappa_p x} \right) \quad (15)$$

$$\text{C, clamped: } u_e = \frac{1}{2} \left(e^{j\kappa_p x} - j e^{-j\kappa_p x} - \frac{\kappa_p}{\kappa_{pj}} (1 - j) e^{-\kappa_{pj} x} \right) \quad (16)$$

$$\text{F, free: } u_e = \frac{1}{2} \left(e^{j\kappa_p x} - j e^{-j\kappa_p x} + \left(\frac{\kappa_p}{\kappa_{pj}} \right)^2 (1 - j) e^{-\kappa_{pj} x} \right) \quad (17)$$

$$\text{G, guided: } u_e = \left(e^{j\kappa_p x} + e^{-j\kappa_p x} \right) \quad (18)$$

Now we know the form of u near the edge and as the area coupling is zero below f_c we calculate the edge coupling from eqs.(9) - (12). Compare with the following figure:

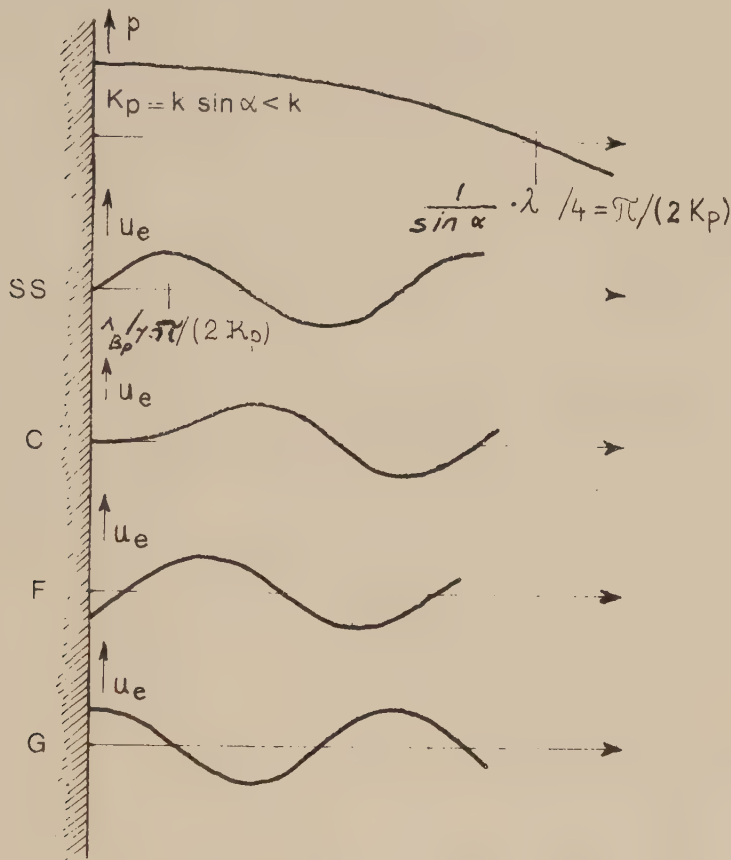


FIGURE 4.9.2. The edge coupling forms.

There will be area cancellation and furthermore $k_p \gg \kappa_p$, so p is almost constant for many u -periods. With $\delta \ll 1$ introduced to give zero at infinity the approximate coupling integrals are:

$$w = \left| \int_0^{\infty} e^{-\delta x} \cdot u_e(x) dx \right| \quad (19)$$

$$w_{SS} = \frac{1}{2} \left| \frac{2}{j\kappa_p} \right| = \frac{1}{\kappa_p} \quad (20)$$

$$w_C = \frac{1}{2} \left| \frac{1-j}{j\kappa_p} + \frac{\kappa_p}{\kappa_{pj}} \cdot \frac{1-j}{\kappa_{pj}} \right| = \frac{1}{\sqrt{2} \kappa_p} \left(1 + \left(\frac{\kappa_p}{\kappa_{pj}} \right)^2 \right) \quad (21)$$

$$\begin{aligned}
 w_F &= \frac{1}{2} \left| \frac{1-j}{j\kappa_p} - \frac{1}{2} \left(\frac{\kappa_p}{\kappa_{pj}} \right)^2 \frac{(1-j)}{\kappa_{pj}} \right| = \\
 &= \frac{1}{\sqrt{2} \kappa_p} \left(1 - \left(\frac{\kappa_p}{\kappa_{pj}} \right)^3 \right)
 \end{aligned} \tag{22}$$

$$w_G = \left| \frac{1}{j\kappa_p} - \frac{1}{j\kappa_p} \right| = 0 \tag{23}$$

The w -values have the dimension of length and can be regarded as the width of a coupling strip around a plate.

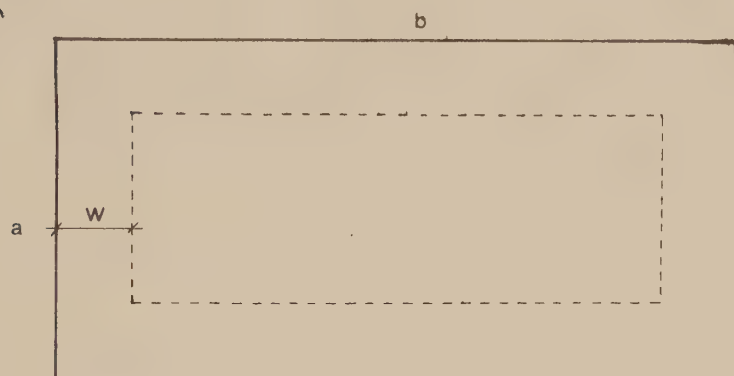


FIGURE 4.9.3. The coupling strip on a plate.

$$Y_C = 2Y_{SS} \quad (8)$$

This simple result has been given in [5] for the range up to f_c . The coupling wall - air is the same as air - wall, so we get $(1/\kappa a)^4$ and $(1/\kappa b)^4$ for coupling, $\sim ka$ and $\sim kb$ for receiving room mode density. To compare τ_{res} and τ_{mass} we introduce Γ :

$$\begin{aligned} \Gamma_{SS} &= \frac{\tau_{res}}{\tau_{mass}} = \left(\frac{2}{\kappa} \right)^4 \left(\frac{\kappa a}{a^4} \cdot \frac{ka}{2} + \frac{\kappa b}{b^4} \cdot \frac{kb}{2} \right) \cdot \frac{1}{4\eta} = \\ &= \frac{2}{\kappa^3} \left(\frac{1}{a^2} + \frac{1}{b^2} \right) \frac{k}{\eta} = \frac{2}{\eta} \left(\frac{1}{a^2} + \frac{1}{b^2} \right) \frac{1}{k_c^2} \sqrt{f/f_c} \end{aligned} \quad (9)$$

$$\Gamma_C = \Gamma_{SS} \cdot 4 \quad (10)$$

The factors $ka/2$ and $kb/2$ are the quotients between one- and two-dimensional mode densities in the receiving room (cf. eq.(4.3.15) to eq.(4.3.17)). The mass law area coupling has only one dimension left and the edge-coupled field two dimensions left for the mode density:

$$\text{b-edge: } \frac{kad}{2\pi} / \frac{d}{\pi} = \frac{ka}{2} \quad (11)$$

$$\text{a-edge: } \frac{kb}{2} \quad (12)$$

In practice we have often observed that the masslaw predicts the transmission loss rather good, but fails to predict the vibration of the partition. The reason is that the coupling factor $(1/ka)$ or $(1/kb)$ (both $\ll 1$) comes in in first degree in Y and in second degree in Γ . Hence it is possible that $\Gamma \ll 1$ and $Y > 1$. As an example we choose $a \approx b \approx 3$ and $k_c \approx 10$ (500 Hz). With $\eta = 0.03$ we get

$$Y_{SS} \approx 4 \sqrt{f/f_c} \quad (13)$$

$$\Gamma_{SS} \approx 0.1 \sqrt{f/f_c} \quad (14)$$

These results are valid when $f \ll f_c$, which gives $\kappa_p \approx \kappa$. According to eq.(4.9.7) we get a tendency of decreasing κ_p and increasing κ_{pj} , as f increases. We shall calculate a mean coupling for α going from 0 to $\pi/2$.

In the SS-case we have for the vibration of the plate $1/\kappa_p$ and for the transmission $1/\kappa_p^2$:

$$\begin{aligned} \frac{Y_{\kappa p}}{Y_{\kappa}} &\approx \int_0^{\pi/2} \left(\frac{\kappa}{\kappa_p} \right) \sin \alpha \, d\alpha = \int_0^{\pi/2} \frac{\sin \alpha}{\sqrt{1 - (f/f_c) \cos^2 \alpha}} \, d\alpha = \\ &= \sqrt{f_c/f} \arcsin \sqrt{f/f_c} \end{aligned} \quad (15)$$

$$\frac{\Gamma_{\kappa p}}{\Gamma_{\kappa}} = \int_0^{\pi/2} (\kappa/\kappa_p)^3 \sin \alpha \, d\alpha = \int_0^{\pi/2} \frac{\sin \alpha}{((1 - (f/f_c) \cos^2 \alpha)^{3/2}} \, d\alpha \quad (16)$$

These integrals are > 1 , which could also be expected, as the mean coupling strip is wider than $1/\kappa$. For $f/f_c = 0.5$ we obtain a factor from (15), 1.1 (0.5 dB). (16) gives about 1.5 dB. The angle limit of β in eq.(4.9.9) is β_g :

$$\beta_g = \arcsin \sqrt{f/f_c} \quad (17)$$

Evidently the variation compared to the approximate $\kappa_p \approx \kappa$ is only of importance when $f \rightarrow f_c$.

For the clamped boundary the coupling is somewhat more complicated. For the transmission we get the following expression with $\cos \phi = \delta$:

$$\frac{\Gamma_{\kappa p}}{\Gamma_{\kappa}} = \int_0^1 \frac{\left(\frac{1}{2} \left(1 - \frac{1 - \delta^2 f/f_c}{1 + \delta^2 f/f_c} \right) \right)^4}{(1 - \delta f/f_c)^{3/2}} \, d\delta \quad (18)$$

As the nearfield $(\kappa_{pj} = 1 + \delta^2 f/f_c)$ increases with δ , (18) will give a

smaller correction to the C-case than (14) to the SS-case.

Rather small corrections for $f < f_c/2$ are shown here for a simple three-dimensional model compared to $\kappa_p \approx \kappa$. A beam model will have $\kappa_p = \kappa$. This is the explanation to the fairly good agreement shown in [5] for a beam model that is used for plates. The estimated TL in [5] at $f/f_c = 1/2$ seems to be somewhat high (see e.g. p.4.10 in [5]). Perhaps this could be corrected the way shown here by eqs.(14) and (16).

The introduced model is simple because we only use one edge at a time and semi-infinite plate to obtain the coupling. In my opinion this is the best way to get an initial correction to the Cremer theory for infinite plates. Thus it is shown that it is not necessary to use complicated models to get a suitable, approximate edge correction for plate vibration and transmission loss.

In [13] is discussed the radiation from panels driven by vibrations and airfields. The radiation is calculated from the spatial correlation of the vibrational field. This is a way to get the driving conditions into the picture, but it is only true if the edge radiation is of minor importance.

For SS we use (3) and (5):

$$D_{SS} = - \frac{\frac{k_p^2}{2} + \frac{\kappa_{pj}^2}{2}}{\kappa_p^2 + \kappa_{pj}^2} \quad (6)$$

$$k \ll \kappa_p \rightarrow \kappa = \kappa_p \approx \kappa_{pj} \rightarrow D_{SS} = \frac{1}{2} \quad (7)$$

For a clamped plate we use (3) and (4):

$$D_C = \frac{-\kappa_{pj}}{-j\kappa_p + \kappa_{pj}} \quad (8)$$

$$k \ll \kappa \rightarrow |D_C| = \frac{1}{\sqrt{2}} \quad (9)$$

As the area mean square of the mass field is 1/2 of the edge mean square and the group velocity for power transmission is $2C_B$ we get:

$$Y = \frac{\text{rev.field}}{\text{massfield}} = \frac{2C_B(a+b)^2}{\omega\eta S} \cdot \frac{D^2}{1/2} \quad (10)$$

The PE-balance giving this is clearly shown below:

$$2\mu_v 2_{\text{mass}} \cdot D^2 \cdot (\text{edge length}) \cdot 2C_B = \mu_v 2_{\text{rev}} \cdot \omega\eta S \quad (11)$$

We now refer to the resonant field as the reverberant field, to distinguish between the MSC-model in par. 4.9 and 4.10 and the PE-method in this paragraph. From (10) we obtain:

$$Y_{SS} = \frac{2}{\kappa\eta} \left(\frac{1}{a} + \frac{1}{b} \right) \quad (12)$$

$$Y_C = Y_{SS} \cdot \frac{D_C}{D_{SS}} = 2Y_{SS} \quad (13)$$

Once more the PE-method has proved to give the same result as a more complicated MSC-method. This is natural, as also the MSC-method requires pure pressure generator (cf. par. 4.3 and 4.4). The wall vibration does not affect the driving pressure.

The room field is easily obtained if $k \ll \kappa$. Then the radiation from the edge strip is of line radiation type into a quarter cylinder. Hence the radiation resistance R_s from par. 2.2 is

$$R_s = 4 \cdot k\rho c/4 = k\rho c \quad (14)$$

The volume velocities are obtained from the radiating narrow strips $1/\kappa$ and $\sqrt{2}/\kappa$:

$$SS \text{ (volume velocity per length unit): } \mu_{W2} = \mu_{v2a} (1/\kappa)^2 \quad (15)$$

$$C: \mu_{W2} = \mu_{v2a} \quad 2(1/\kappa)^2 \quad (16)$$

(The expressions are similar at an a-edge.)

We use Y and the line radiation compared to the area radiation for the massfield and get:

$$\Gamma_{SS} = \frac{2k}{\kappa^3 \eta} \cdot \frac{\left(\frac{b}{a} + \frac{a}{b}\right)}{ab} \quad k = \frac{2k}{\kappa^3 \eta} \left(\frac{1}{a^2} + \frac{1}{b^2}\right) \quad (17)$$

$$\Gamma_C = 4 \Gamma_{SS} \quad (18)$$

The result is again the same as for the MSC-method, which is due to the fact that pure volume generators are used. The pressure does not affect the driving velocity.

Thus the edge radiation is shown by a simple PE-method involving bending wave scattering, which will be referred to as an SPE-method. The SPE-method is more compatible to the usual Sabine PE-model in the rooms than other methods. It is a favourable model to use in text-books on the

graduate level. It gives an introductory and easily understood correction to the Cremer theory of "infinite" walls.

The radiation factor s can be estimated by the following expression if $Y_{SS} > 1$:

$$s = \frac{1 + \Gamma_{SS}}{1 + Y_{SS}} \quad (19)$$

If $Y_{SS} > 1$ but $s \ll 1$ we confirm the frequently observed, and also in par. 4.10 shown, statement that the masslaw predicts the TL well but not the vibration of the wall. In the example of par. 4.10, s was about $1/40$ (-16 dB).

It is also interesting to study the sign of D . D_{SS} is negative and we always have:

$$D + D_j = -1 \quad (20)$$

The scattering gives a primary wave that diminishes the field and the radiation if η is large. In that case the reverberant field is attenuated in some wavelength. With high damping, multiple layers used in noise abatement η can be very large. If η is small we get an amplified vibration by $1/\eta$. The two cases are illustrated in FIG 4.11.1, which shows both diminished and amplified field.

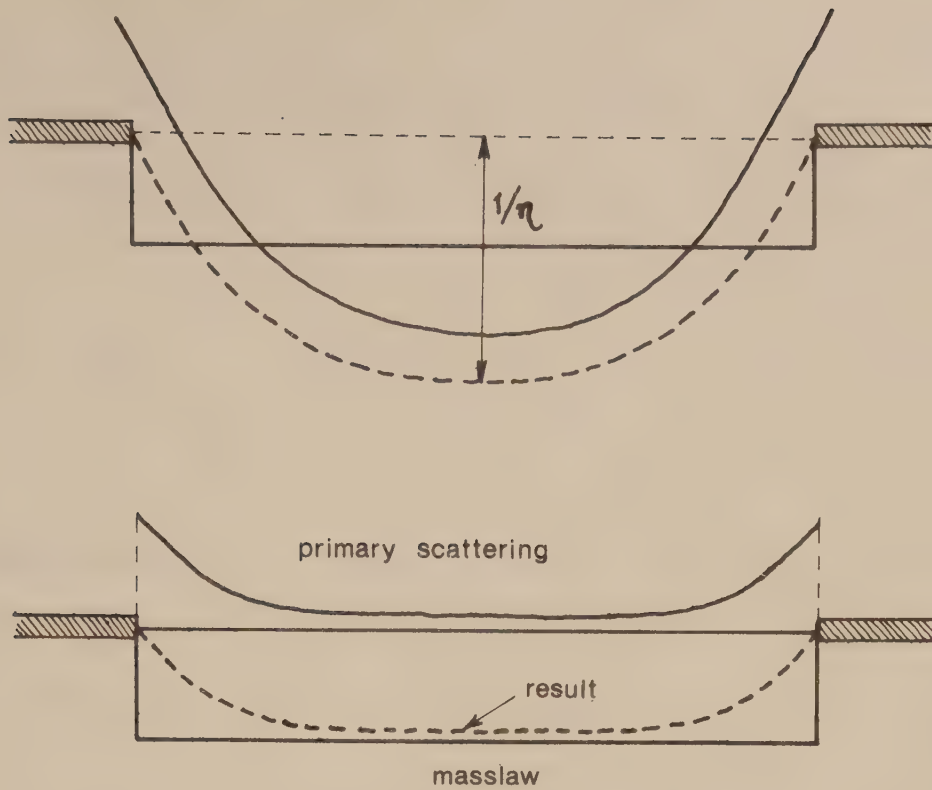


FIGURE 4.11.1. The scattered field.

The SPE-model has not the SEA energy balance, which makes SPE suitable when also a non-resonant energy path is present. It can be given the following energy network:

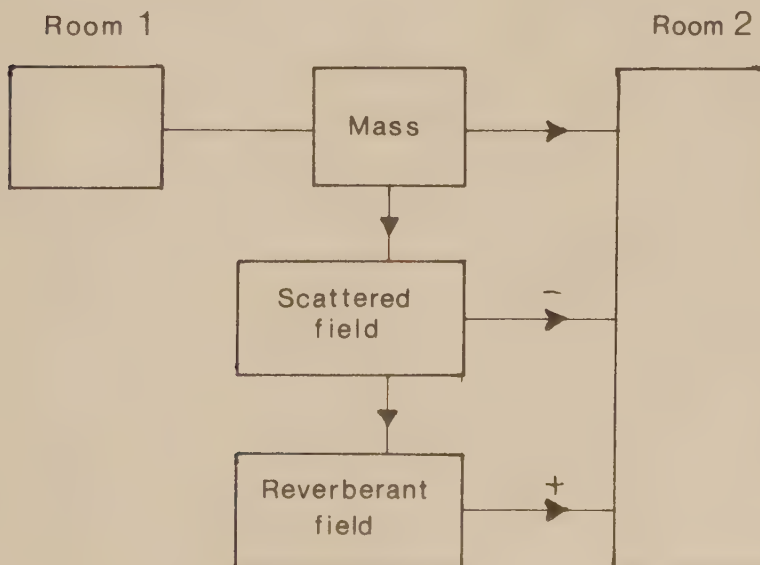


FIGURE 4.11.2. The transmission paths.

4.12. The half-infinite plate at coincidence

Now we shall study the growth of a bending wave at coincidence. We consider the following situation:



FIGURE 4.12.1. The growing bending wave.

We are particularly interested in the case at the coincidence frequency, f_ϕ , which gives:

$$k(f_\phi) \sin \phi = \kappa(f_\phi) \quad (1)$$

We shall study the growth of x of the outgoing bending wave. First we shall look at the time growth of the vibration of a simple resonant system driven at resonance.

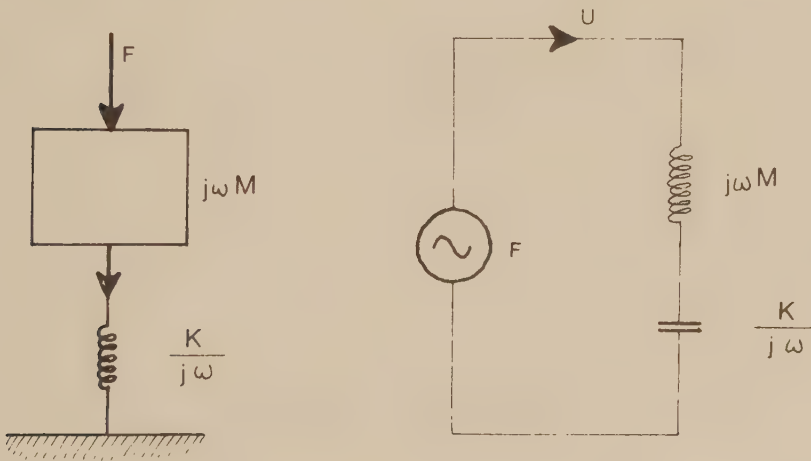


FIGURE 4.12.2. The resonant system.

The transfer function is

$$\frac{u}{F} = \frac{1}{j\omega M + K/j\omega} = \frac{j\omega}{\omega_o^2 - \omega^2} \quad (2)$$

$$\omega_o^2 = K/M \quad (3)$$

We start a sinusoidal force at $t = 0$:

$$F_o \sin \omega t \quad (4)$$

The Fourier transform of (4) is

$$F_\Omega = \frac{F_o \Omega}{\Omega^2 - \omega^2} \quad (5)$$

The "time coincidence" is obtained for $\Omega = \omega_o$ and we get

$$u(\omega) = \frac{F_o \omega_o}{M(\omega_o^2 - \omega^2)^2} \quad (6)$$

The inverse transform will be

$$v(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{F_o \omega_o^2 e^{j\omega t}}{M(\omega_o^2 - \omega^2)^2} d\omega \quad (7)$$

The second order pole $\omega = \omega_o$ gives the following residue:

$$\frac{\partial}{\partial \omega} \cdot \frac{e^{j\omega t}}{(\omega_o + \omega)^2} = \frac{e^{j\omega t}}{(\omega_o + \omega)^2} \left(jt - \frac{2}{\omega_o + \omega} \right) \quad (8)$$

The time function is obtained from (7) and (8):

$$v(t) = \frac{F}{2M} \left(t - \frac{j}{\omega_o} \right) \sin(\omega_o t) \quad (9)$$

For $t \gg 1/\omega_0$ we get the following linearly growing vibration:

$$v(t) \approx \frac{Ft}{2M} \sin(\omega_0 t) = \frac{F}{\omega_0 M} \cdot \frac{\omega_0 t}{2} \sin(\omega_0 t) \quad (10)$$

With the losses η_1 and $\eta_1 \ll 1$ we modify ω_0 and get first order poles:

$$\bar{\omega}_0^2 = \omega_0^2 (1 - j\eta_1) \quad (11)$$

The inverse transform is

$$\begin{aligned} v(t) &= \frac{1}{2\pi} \int \frac{F_0 \bar{\omega}_0 \omega e^{j\omega t}}{M(\bar{\omega}_0^2 - \omega^2)(\omega_0^2 - \omega^2)} d\omega = \\ &= \frac{F_0}{\omega_0 M \eta_1} \left[\sin(\omega_0 t) - \sin(\omega_0 t) e^{-\omega_0 \eta_1 t/2} \right] = \\ &= \frac{F}{\omega_0 M \eta_1} \sin(\omega_0 t) \left[1 - e^{-\omega_0 \frac{\eta_1}{2} t} \right] \end{aligned} \quad (12)$$

For small η -values we get from the first terms of the Taylor series:

$$v(t) \approx \frac{F_0}{\omega_0 M \eta_1} \omega_0 \frac{\eta_1}{2} t \sin(\omega_0 t) = \{ (10) \} \quad (13)$$

With this simple case as a background we perform the calculations for the bending wave at coincidence. We use the single boundary and a steady state deriving wave according to FIG 4.12.3. To avoid nearfield terms, FIG 4.12.1 is modified. The shielding slab gives a "starting wave" at $x = 0$.



FIGURE 4.12.3.

We get the κ -transform of the driving pressure:

$$p_{\kappa} = p \int_0^{\infty} e^{j\kappa x} \cdot e^{-jkx \sin \phi} \cdot dx = \frac{P}{j(\kappa - k \sin \phi)} \quad (14)$$

With $\kappa_{\phi}^4 = \omega^2 M/B$ we get for the bending wave:

$$u_{\kappa} = p_{\kappa} \frac{j\omega}{B(\kappa_{\phi}^4 - \kappa^4)} \quad (15)$$

At coincidence $k \sin \phi = \kappa_{\phi}$ and we get

$$u_{\kappa} = p \frac{j\omega}{B(\kappa_{\phi}^4 - \kappa^4)(\kappa - \kappa_{\phi})} = p \frac{j\omega}{(\kappa_{\phi}^2 + \kappa^2)(\kappa_{\phi} + \kappa)(\kappa_{\phi} - \kappa)^2} \quad (16)$$

$$u_{\phi}(x) = \frac{pj\omega}{B} \int \frac{e^{-j\kappa x}}{(\kappa_{\phi}^2 + \kappa^2)(\kappa_{\phi} + \kappa)(\kappa_{\phi} - \kappa)^2} d\kappa \quad (17)$$

The residue of the second order pole $\kappa = \kappa_0$ is the first derivate $\partial/\partial\kappa$:

$$\frac{e^{-j\kappa x}}{(\kappa_{\phi}^2 + \kappa^2)(\kappa_{\phi} + \kappa)} \left(\frac{2\kappa}{(\kappa_{\phi}^2 + \kappa^2)} + \frac{1}{\kappa_{\phi} + \kappa} - jx \right) \quad (18)$$

(18) and (17) give u_x in the following form:

$$u_\phi(x) = \frac{e^{-j\kappa_\phi x}}{4\kappa_\phi^3} \left(\frac{3}{2\kappa_\phi} - jx \right) \frac{p\omega}{B} = \frac{p}{\omega M} \cdot \frac{e^{-j\kappa_\phi x}}{4} (\kappa_\phi x + 3j/2) \quad (19)$$

On a limited surface a good approximation is thus that the coincidence increases the primary wave by $\kappa_\phi x_1/4$ compared to the masslaw:

$$\frac{u_{\text{prim}}}{u_{\text{mass}}} \approx \kappa_\phi x_1/4 \quad (20)$$

By a loss factor η_1 we take into account the solid friction and the load from radiated sound fields:

$$\bar{\kappa}_\phi^4 = \kappa_\phi^4 (1 - j\eta_1) \quad (21)$$

The driving wave is not attenuated, so $k \sin \phi$ is still $= \kappa_\phi$ at coincidence. Now we get only first order poles and the following inverse transform:

$$\begin{aligned} u(x) &= \frac{pj\omega}{B} \int \frac{e^{-j\kappa x}}{(\bar{\kappa}_\phi^4 - \kappa^4)(\kappa_\phi - \kappa)} d\kappa = \\ &= \frac{P}{\omega M \eta_1} e^{-j\kappa_\phi x} \left[1 - e^{-\kappa_\phi x \eta_1/4} \right] \end{aligned} \quad (22)$$

For small $\eta\kappa_\phi x$ -values we obtain:

$$u(x) = \frac{P}{j\omega M} e^{-j\kappa_\phi x} \cdot \frac{\kappa_\phi x}{4} \quad (23)$$

The wave grows towards a final value obtained from η_1 , but the first part is independent of η_1 and is the same as with $\eta_1 = 0$.

4.13. The coupling and edge scattering at f_ϕ and f_c

For frequencies above the critical frequency there is area coupling from a diffuse field to resonant modes of the wall. These terms couple to the receiving field as well.

For one-directional incident waves the area coupling is present for equal κ -numbers along the surface (coincidence):

$$\kappa = k \sqrt{f_c/f} = k \sin \phi \quad (1)$$

$$f_\phi = f_c / \sin^2 \phi \quad (2)$$

At f_ϕ we expect a dip in the TL-curve because of perfect coupling and resonance. However, there is occasionally a TL-dip also at f_c .

In [46] measurements are shown on a window with one 4 mm pane, where the dip at f_c is very obvious. The curves are represented here:

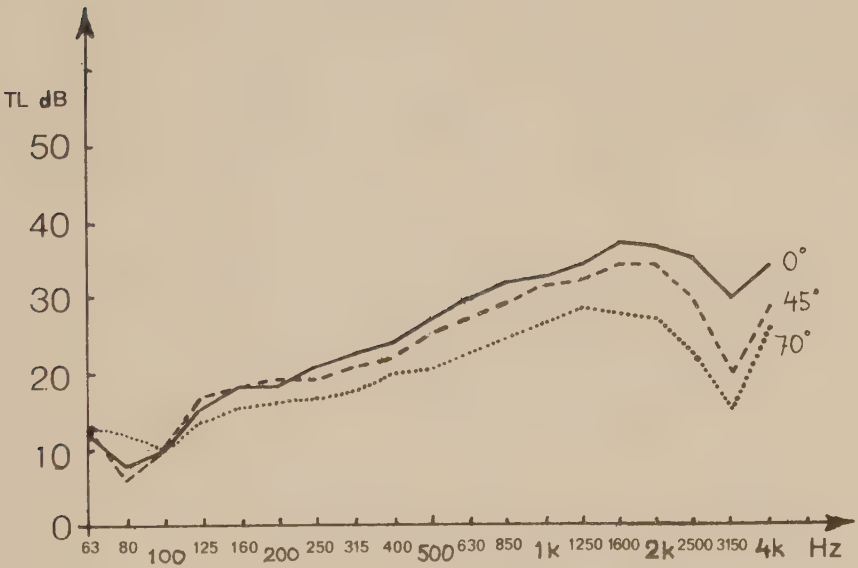


FIGURE 4.13.1. A 4 mm pane, from [46].

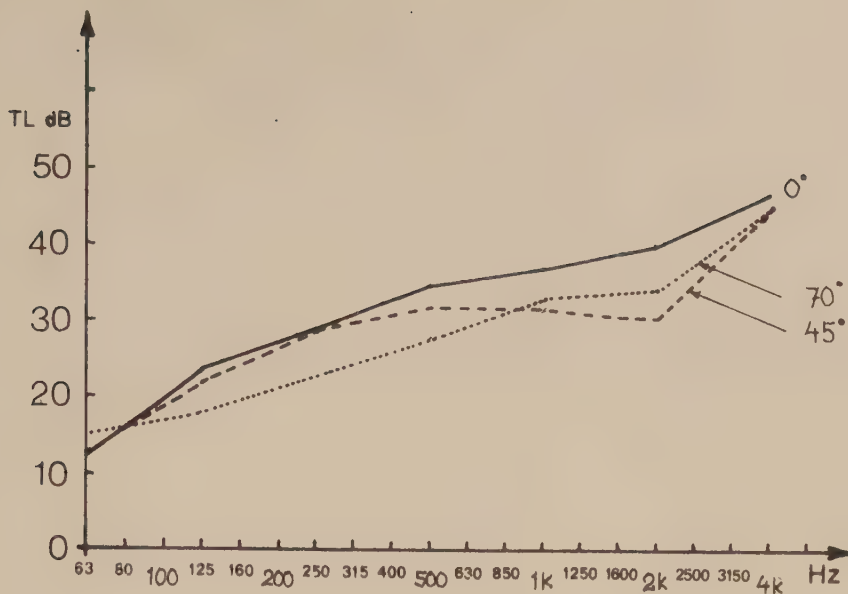


FIGURE 4.13.2. A 12 mm pane from [46].

For $f_c < f < f_\phi$ we have edge-scattered vibrational fields as for $f < f_c$. The radiation is however of area type if $f > f_c$. The radiation is particularly efficient at $f \sim f_c$, grazing radiation.

At f_ϕ we get area coupling and radiation. It is common to use the infinite plate solution and replace the inner losses η_1 by the combined loss factor η for plate and boundaries.

The coupling method is tedious to use, so I have tried to adopt the SPE-model also for $f > f_c$.

One might think that the infinite plate model is not valid for small partitions. However, with η_1 changed for η the model can be used also in this case. There is also an interesting case for partitions not small enough to use only η and not large enough to use only η_1 . Both η_1 and η have to be used in such a case.

From par. 4.12 we get u_ϕ for a pure tone wave at coincidence. We refer to that value as $(1/(\omega M \beta))^2$. For a noiseband (filter band B_F) of course we get less vibration. We integrate over the peak at f_ϕ to get a mean square response.

$$\frac{\overline{v}_{\text{prim}}^2}{\overline{v}_q^2} = \frac{\int_{f_1}^{f_2} |H|^2 df}{B_F} = \frac{1}{B_F} \int_{f_1}^{f_2} \frac{1}{(\omega M)^2 (\beta^2 + (1 - (f/f_\phi)^2)^2)} df =$$

$$= \frac{1}{(\omega M)^2} \cdot \frac{f_\phi \pi}{B_F \cdot 2} \cdot \frac{1}{\beta} = \frac{1}{(\omega M)^2} \cdot \frac{1}{\Delta_B} \cdot \frac{1}{\beta} \quad (3)$$

$\Delta_B = B_F / (f_\phi \pi / 2)$, relative filter bandwidth.

We obtain a reverberant field using the same method as for $f < f_c$, but with the edge reflexion = 1. The power balance in the reverberant field gives:

$$\overline{v}_{\text{rev}}^2 = \frac{2C_B \overline{v}_{\text{prim}}^2 (a + b)^2}{\omega \eta a b} \quad (4)$$

For comparison with the masslaw we use Y_ϕ , the reverberant compared to the masslaw vibration. For a small wall with $1/\beta = \kappa_\phi \sqrt{ab} / 4$ or $\kappa_\phi \cdot a/4$ and $\kappa_\phi \cdot b/4$ in the two directions we obtain:

$$Y_\phi = \frac{1}{\Delta_B} \cdot \frac{4\kappa \left(\frac{ab}{4} + \frac{ba}{4} \right)}{2 \kappa a b \eta} = \frac{1}{\Delta_B \eta} \quad (5)$$

For an infinite plate we get the usual Cremer expression:

$$\begin{aligned} \overline{v}^2 &= \frac{1}{B_F} \cdot \frac{\overline{q}^2}{(\omega M)^2} \int_{f_1}^{f_2} \frac{1}{\eta_1^2 + (1 - (f/f_\phi)^2)^2} df = \\ &= \left(\frac{1}{\omega M} \right)^2 \cdot \overline{q}^2 \cdot \frac{1}{\Delta_B \eta_1} \end{aligned} \quad (6)$$

The expression (5) is thus similar to the result for an infinite plate, where only the primary field is present. In usual calculations η_1 is replaced by the combined η . However, the solution shown here will give a different result if $\kappa l/4 \gg 1/\eta_1$. From eq.(4.12.23) we obtain:

$$Y_\phi = \frac{1}{\Delta_B \eta_1} \cdot \frac{2C_B \cdot (a + b)}{\omega a b \eta} = \frac{2 \left(\frac{1}{a} + \frac{1}{b} \right)}{\Delta_B \eta_1 \kappa \eta} \quad (7)$$

If we include the primary field (7) is modified:

$$Y_{\phi} = \frac{1}{\Delta_B \eta_1} \left(1 + \frac{2 \left(\frac{1}{a} + \frac{1}{b} \right)}{\kappa \eta} \right) \quad (8)$$

Finally we also get a solution from eq.(4.12.22) for partitions between the very large (7) and the small (5):

$$Y_{\phi} \approx \frac{1}{\Delta_B \eta_1} (1 - \exp(-\kappa \sqrt{ab} \eta_1/4)) \cdot \frac{2}{\kappa \sqrt{ab} \eta} \quad (9)$$

The radiation at f_{ϕ} is the same as for a massfield, so we obtain:

$$\Gamma_{\phi} = Y_{\phi} \quad (10)$$

We shall now move from f_{ϕ} to f_c . If $f_{\phi} > f_c$, as before we get a scattered field from the primary massfield at f_c :

$$Y_c = \frac{1}{2} \cdot \frac{2C_B \cdot 2(a+b)}{\omega_c \eta(ab)} = \frac{2}{\kappa_c \eta} \left(\frac{1}{a} + \frac{1}{b} \right) \quad (11)$$

Compared to the edge radiation and even to the case at f_{ϕ} the radiation will be more efficient at f_c . This is due to the almost grazing area radiation with the following radiation resistance:

$$R_s \approx \frac{\rho c}{\sqrt{1 - (\kappa/k)^2} + 4/(\kappa l)} \quad (12)$$

For third-octave bands and octave bands $4/(\kappa l)$ can as a first approximation be omitted in the integration.

With f_c at the lower limit the radiation factor for the band $(f_c, (1+\delta)f_c)$:

$$s = \frac{1}{f_c \delta} \int_{f_c}^{f_c(1+\delta)} \frac{1}{\sqrt{1 - (f_c/f)^2}} df = \frac{1}{\delta} \int_1^{1+\delta} \frac{1}{\sqrt{1 - 1/\xi^2}} d\xi \approx$$

$$\approx \frac{1}{\delta} \int_0^{\delta} \frac{1}{\sqrt{\Delta\xi}} d\Delta\xi = \frac{1}{\delta} 2\sqrt{\delta} = \frac{2}{\sqrt{\delta}} \quad (13)$$

From this we get Γ_c for TL-comparisons with the masslaw:

$$\Gamma_c = \frac{4}{\kappa_c \eta \sqrt{\delta}} \left(\frac{1}{a} + \frac{1}{b} \right) \quad (14)$$

Γ_c is often > 1 . As an example we choose $\eta = 0.01$, $\kappa_c = 30$ and $a = b = 1$ m. For $\delta = 0.2$ we get $\Gamma_c \approx 30$ (15 dB) (cf. FIG 4.13.8).

At f_c (narrow band or pure tone) the coupling width $1/\kappa$, small compared to a or b , is compensated by the increased radiation resistance $\rho c(\kappa/f)$ { a or b } for grazing radiation, so we get a simple Γ_c :

$$\Gamma_c(f = f_c) = \frac{4}{\kappa_c \eta} \left(\frac{a}{b} + \frac{b}{a} \right) \frac{\kappa_c}{4} = \frac{2}{\eta} \quad (15)$$

For $\phi = 0$ and at $f = f_c$ we get the following picture of the sound transmission:

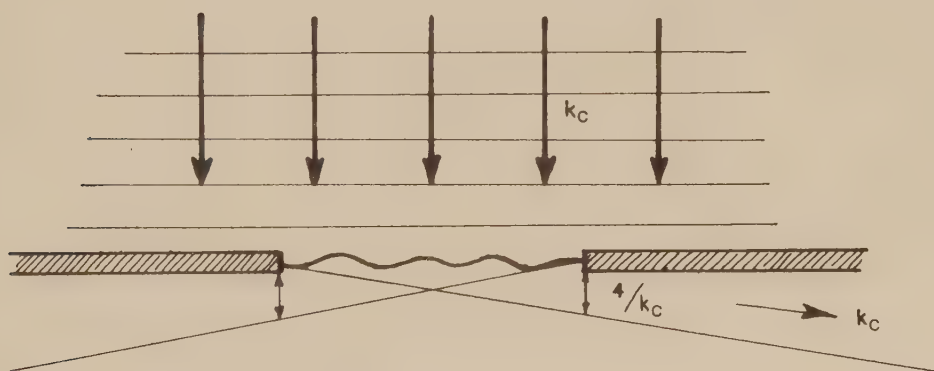


FIGURE 4.13.3. Grazing radiation for normal incidence.

We conclude that there can be a TL-dip not only at f_ϕ but also at f_c , but that the f_ϕ -dip is usually deeper. In cases with a very deep valley at f_c this can be due to a deep niche on the driving side of the window. The niche modes can rearrange the incident sound and increase the coupling at f_c . However, the results in this paragraph show that this is not necessary to get a visible dip at f_c .

I have also performed a model experiment according to the following figures.

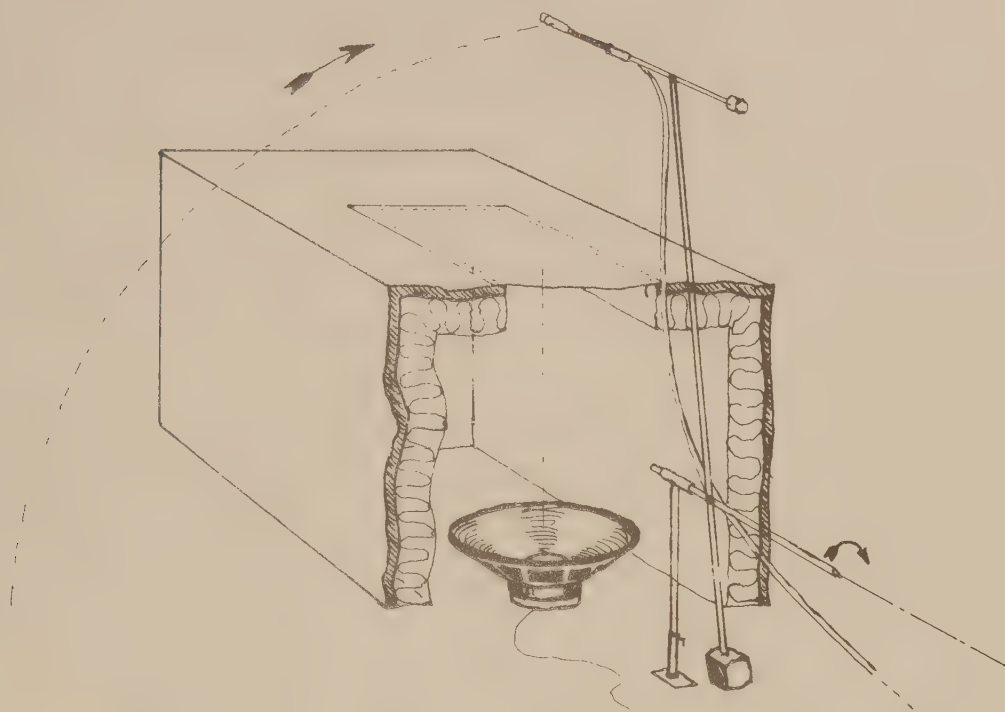


FIGURE 4.13.4. Measurement set-up for radiation at f_c .

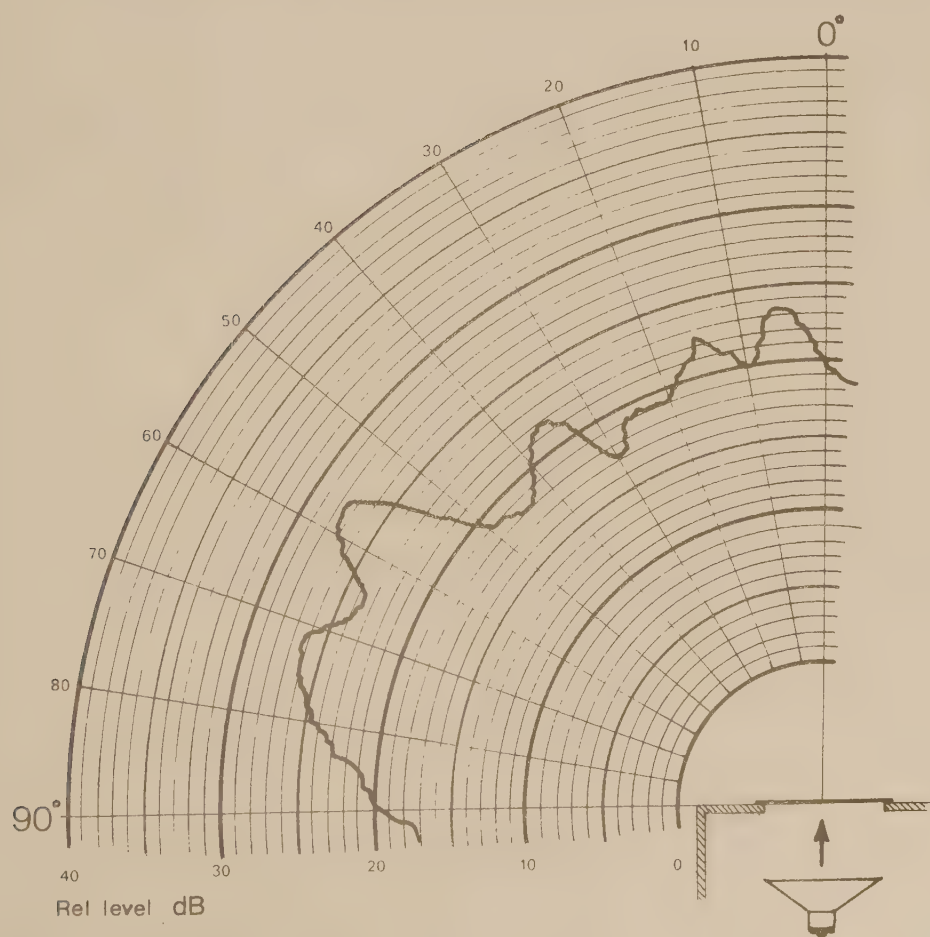


FIGURE 4.13.5. Radiated sound pressure at $f_c \approx 5$ kHz, 2 mm Al, 300 Hz noise.

4.14. The coupling between equal rooms

A case in which the PE-model can give a wrong result is in the TL between two equal rooms with masslaw transmission. The reason is that for equal rooms the acoustical load on the partition is larger, as the resonances of the rooms are identical.

If we have $\Delta_r \gg 1$ and therefore no modal overlap, the response of one resonance of the driving room is calculated from the identical mode resonance in the receiving room.

We get the following network for equal rooms. X is the normalized frequency deviation from the resonance. For simplicity the resonance frequency is put = 1 in the network.

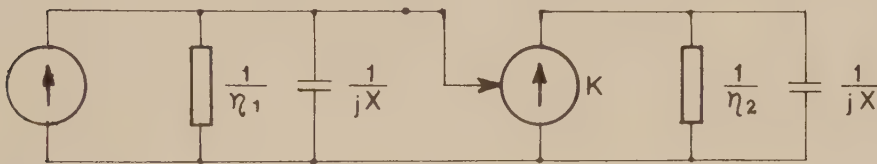


FIGURE 4.14.1. Resonance network.

$$|H_1|^2 \sim \frac{1}{\eta_1^2 + X^2} \quad (1)$$

$$|H_2|^2 = |H_1|^2 \cdot |H'|^2 \sim K \frac{1}{\eta_1^2 + X^2} \cdot \frac{1}{\eta_2^2 + X^2} \quad (2)$$

The mean squares are obtained from the integral responses, which are:

$$\int |H_1|^2 = \int_{-\infty}^{\infty} \frac{1}{\eta_1^2 + X^2} dx = \frac{\pi}{\eta_1} \quad (3)$$

We split the denominator and from (2) we get:

$$J_{|H_2|^2} = \int_{-\infty}^{\infty} \frac{K}{(\eta_1 - jX)(\eta_1 + jX)(\eta_2 - jX)(\eta_2 + jX)} dX \quad (4)$$

(4) is evaluated from the residues at $X = j\eta_1$, and $X = j\eta_2$ if the integration path is closed in the upper half-plane.

$$\begin{aligned} J_{|H_2|^2} &= K \cdot \left(\frac{-1}{\eta_1^2 + \eta_2^2} \left(\frac{1}{j2\eta_1} - \frac{1}{j2\eta_2} \right) \right) 2\pi j = \\ &= K \frac{\pi}{(\eta_1 + \eta_2)\eta_1\eta_2} \end{aligned} \quad (5)$$

If the rooms are unequal the mean response is simply coupling to the mean response of room 2, $1/(\eta_2^2 \Delta_r)$:

$$J_{|H_3|^2} = K \cdot \frac{\pi}{\eta_1} \cdot \frac{1}{\eta_2^2} \cdot \frac{1}{\Delta_{r2}} \quad (6)$$

$$\frac{\text{Equal}}{\text{Unequal}} = \frac{\eta_2 \Delta_{r2}}{\eta_1 + \eta_2} \quad (7)$$

The used model breaks down if $\Delta_{r2} < 2$ for (5), as there will be coupling also to other modes than the identical one. This leads to a larger value than (5).

If $\Delta_{r2} > 2$ and $\eta_2 \approx \eta_1$ the effect of equal rooms is a lower TL than for unequal rooms.

This type of deduction has been used in [5] both for $\Delta_{r2} > 2$ and < 2 . The latter case leads to higher TL for equal rooms. This remarkable result has been supported by experiments. However, these experiments have not been performed with equal and unequal rooms, but by varying the diffusors.

4.15. Concluding remarks to Part 4

The discussion about the use of different methods to obtain the mean square pressure or energy of a room is important to the choice of simplifying methods. The shown influence of source impedance should be observed for the current discussions about noise power measurements.

The presented simple power energy methods are claimed to fill a need in practice and education.

The comparisons of the TL at critical and coincidence frequency show how and why the simple Cremer theories can be used also for small partitions.

My hope is that most of the results in Part 4 can be adopted soon, to improve the education in architectural acoustics.

PART 5

SOUND PROPAGATION OVER AN ABSORBING PLANE

5.1. Introduction

In this part the main interest is devoted to the sound propagation from a point source at or near a plane, locally reacting surface.

The transform of a point source into line source waves is used for a point source at the surface. This shows the space wave but also a surface wave as a resonance in the connected radiation network. The power transfer to the different types of waves is computed.

The radiation idea is used also for a point source in the space. The two-dimensional k -transform leads to a plane radiating source layer. Rewriting of this solution leads to the angular spectrum, which is shown to be a secondary form of the layer transform.

Sommerfeldt's method [48] is compared to Weyl's [49]. The latter solution is also used by Ingård [50]. A discussion about the missing surface wave concerns the integration paths and the boundary conditions for the different methods. This is meant as a complement to Wenzel's [47] new solution of the surface waves.

The main purpose is also to show that a feeling for the physical problem behind a transform is necessary to avoid mistakes at the application of the method.

5.2. The point source in a plane with surface admittance $\neq 0$

We shall adopt the radiation method to a resilient surface. In Part 2 the radiation from a point source in a hard plane was calculated. If the surface has an admittance $v/\rho c$, part of the velocity is "shunted" by v and not acting on the produced field. The network in FIG 5.2.1 is valid for the velocity distribution $u_k(\kappa)$ (cf. Part 2). From the point source of the volume velocity W we get:

$$u_k(\kappa) = \iint W(x, y) \cdot e^{+j(\kappa_x x + \kappa_y y)} dy dx \quad (1)$$

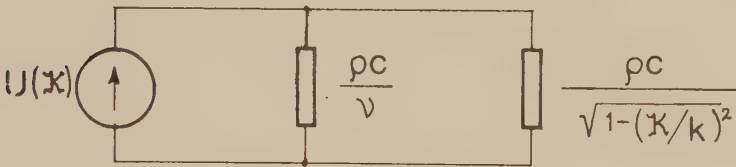


FIGURE 5.2.1. The effect of a v -layer. Specific impedance network.

The effect of v is the same if $u(\kappa)$ is regarded as a "vibrating plate" below the v -layer or as a v -layer on a hard resting surface with $\delta v(\kappa)$ as a Dirac- δ velocity layer above v (cf. Part 1).

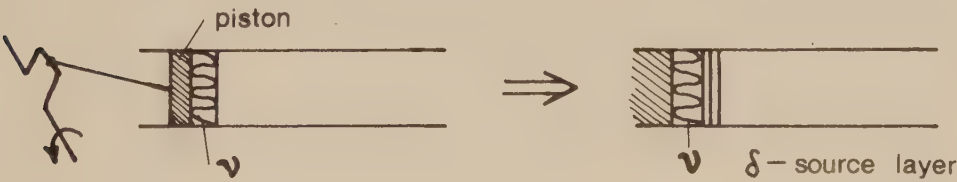


FIGURE 5.2.2. The δ -layer.

The point source can be a source in the hard surface or just above the v -layer. This is illustrated by an acoustical impedance network according to FIG 5.2.3.

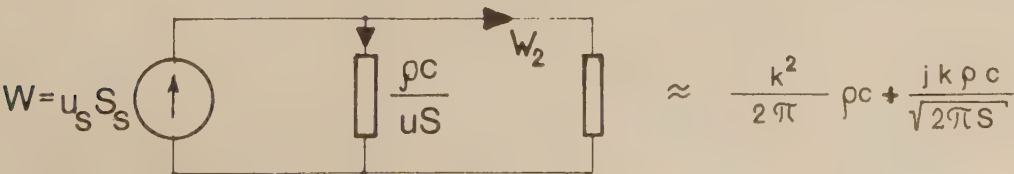


FIGURE 5.2.3. The point source network. Volume velocity impedance network.

As the source area goes to zero, $\sqrt{S}/S \rightarrow \infty$. Hence $W_2 \rightarrow W$ and the radiation technique used for sources in the hard plane can be used for a point source just above the v -layer.

Hence the propagation from a point source is equivalent to a radiation problem. From the network we directly get the pressure of an outgoing wave, originally deduced to fulfil the boundary conditions.

$$p = \frac{u_\kappa}{v + \sqrt{1 - \xi^2}} \quad (2)$$

and

$$k_z = k\sqrt{1 - \xi^2}$$

$$\kappa_x = \kappa \cos \alpha; \quad \kappa/k = \xi \quad (3)$$

$$\kappa_y = \kappa \sin \alpha$$

For simplicity we put $y = 0$ as before. This is no limitation at all for the solution. We get the total pressure as an integral of all $v(\kappa)$ -radiated waves.

The reason for the simple radiation formulas is that the transform is matched to the boundary type. A plane infinite boundary demands transform on an infinite plane. See also the discussion of radiation in Part 2.

Then the pressure in a point $(x, 0, z)$ will be $p(x, z)$:

$$p(x, z) = \frac{Wk^2 \rho c}{2\pi} \int_0^{2\pi} \int_0^\infty \frac{e^{-j(k\xi x \cos \alpha + k\sqrt{1 - \xi^2} z)}}{v + \sqrt{1 - \xi^2}} \xi d\xi \quad (4)$$

The α -integral involves $\exp(-jk\xi x \cos \alpha)$ but can be performed if we use the Bessel series. We put $k\xi x = \beta$ and get:

$$e^{-j\beta \cos \alpha} = J_0(\beta) + 2 \sum J_{2n}(\beta) \cos(2n\alpha) -$$

$$- j 2 \sum J_{2n-1}(\beta) \sin((2n-1)\alpha) \quad (5)$$

All terms except J_0 cancel out on $(0, 2)$, so we get:

$$p = \frac{Wk^2 \rho c}{2\pi} \int_0^\infty \frac{e^{-jk\sqrt{1-\xi^2}z}}{v + \sqrt{1-\xi^2}} J_0(kx\xi) \xi d\xi \quad (6)$$

From the radiation method we have thus obtained an expression for the propagation from a point source in the v -plane. Later we shall use the same plane two-dimensional transform into a δ -source layer in free space and above a v -surface.

To get outgoing waves for $\xi < 1$ and attenuated (not increasing) waves for $\xi > 1$ we have to define $\sqrt{1-\xi^2}$ as follows:

$$\zeta = \sqrt{1-\xi^2} = \begin{cases} \sqrt{1-\xi^2}, & \xi \leq 1 \\ -j\sqrt{\xi^2-1}, & \xi \geq 1 \end{cases} \quad (7)$$

The factor $v + \sqrt{1-\xi^2}$ can give poles at ξ_v with ξ_v from the following connection:

$$\xi_v = -\sqrt{1-v^2} \quad (8)$$

$$\zeta_v = -v$$

If v is purely imaginary the pole is on the axis $\text{Re}(\xi) > 1$ and on the imaginary ζ -axis:

$$\xi_v = \sqrt{1 + (\text{Im}(v))^2} \quad (9)$$

$$-\zeta_v = j(\text{Im}v)$$

$\text{Im}(v)$ must be positive to give a pole on the chosen branch $-j\sqrt{\xi^2-1}$. A pole on the $\text{Re}(\xi)$ -axis gives a Cauchy principal value that will be a real increment to the power.

For $v = 0$ the only real power comes from the integral $(0, 1)$ (cf. par. 2.2) as $(1, \infty)$ is imaginary. Now we include the latter part as v can give a pole and real power from the integral part $(1, \infty)$.

The v -type, which gives a pole on the physically chosen branch $+j\sqrt{\xi^2 - 1}$, is of compliance type. This type is obtained with thin absorbent layers.

The radiated power is $\text{Re}(W_{p_0})$. p_0 is the pressure at the source point and we get:

$$\Pi = \frac{|W|^2 k^2 \rho c}{2\pi} \text{Re} \int_0^\infty \frac{1}{v + \sqrt{1 - \xi^2}} \xi d\xi \quad (10)$$

Because of the sign choice, which must be performed at $\xi = 1$, we divide the integral into $(0, 1)$ and $(1, \infty)$.

$\xi > 1$ gives $\exp(-k\sqrt{\xi^2 - 1} z)$ and therefore attenuation in the z -direction. Only the $(0, 1)$ -part will give unattenuated space waves. We get the corresponding power Π_s in the branch $(\rho c / \sqrt{1 - \xi^2})$ of the network (FIG 5.2.1). Thus the power in Rev included in (10) is omitted. This part is not radiated to the space.

$$\Pi_s = \frac{|W|^2 k^2 \rho c}{2\pi} \int_0^1 \frac{1}{\left|1 + \frac{v}{\sqrt{1 - \xi^2}}\right|^2} \frac{\xi}{\sqrt{1 - \xi^2}} d\xi \quad (11)$$

There cannot be a pole on $(0, 1)$ as $\text{Re} v \geq 0$. Hence the integral value is normally less than for $v = 0$. We rewrite the integral (11) with $\zeta = \sqrt{1 - \xi^2}$:

$$\int_0^1 \frac{1}{(\zeta + \text{Re}(v))^2 + (\text{Im}(v))^2} \zeta^2 d\zeta \quad (12)$$

As a typical value for a grass ground we choose (cf. [7]):

$$v = 0.1 + j 0.3 \quad (13)$$

$$\int_0^1 \frac{1}{(\zeta + 0.1)^2 + 0.3^2} \zeta^2 d\zeta = 0.58 \quad (14)$$

The hard boundary case can be separated, as we have the following identity:

$$\begin{aligned} \frac{1}{\sqrt{1-\xi^2} + v} &= \frac{1}{\sqrt{1-\xi^2}} - \frac{1}{\sqrt{1-\xi^2} + v} \cdot \frac{v}{\sqrt{1-\xi^2}} = \\ &= p_{\text{Khard}} - p_{\text{Kv}} \end{aligned} \quad (15)$$

Now the substitution $\zeta = \sqrt{1-\xi^2}$ gives:

$$p_{\text{KvO}} = \frac{Wk^2 \rho_c}{4\pi} \int_{L=\zeta_\xi} \frac{2v}{\zeta + v} \cdot e^{-jk\zeta z} \cdot J_0(kx\sqrt{1-\zeta^2}) d\zeta \quad (16)$$

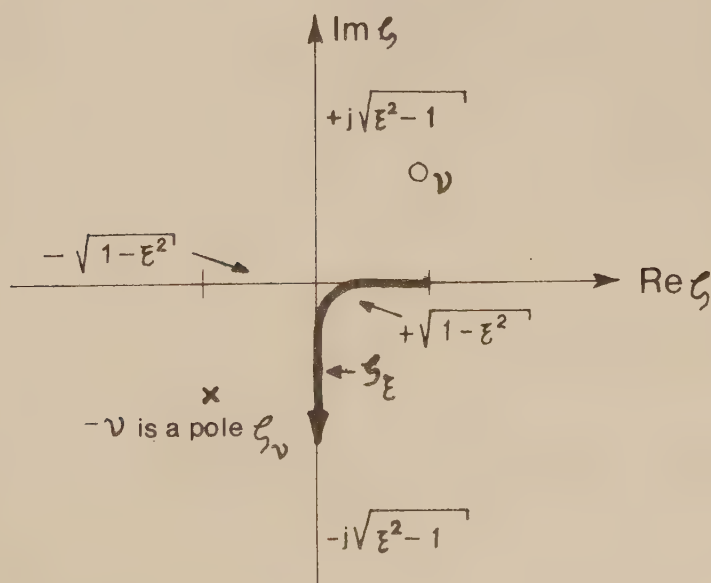


FIGURE 5.2.4. The situation of the possible roots and the path ζ_ξ , that is, the mapping of the real ξ -axis with the chosen sign.

5.3. Some characteristics of propagation curves

The most obvious ground effect on the propagation over a soft lawn is the very strong attenuation at certain frequencies. The following curve from [7] with loudspeaker and microphone on the ground shows a level dip of 30 dB. The distance is 10 m, the centre of the membranes about 5 cm above ground.

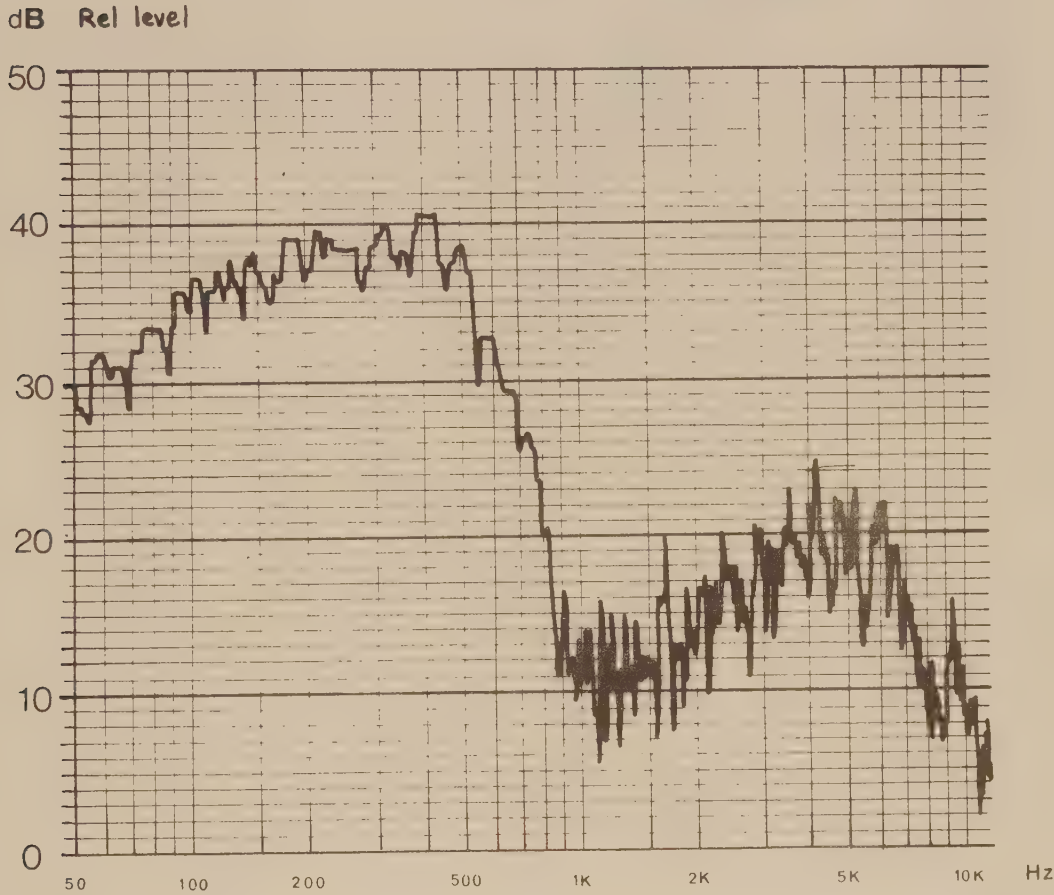


FIGURE 5.3.1. Propagation curve with loudspeaker and microphone on the ground.

With a membrane centre 0.23 m above the ground the following curve is obtained:

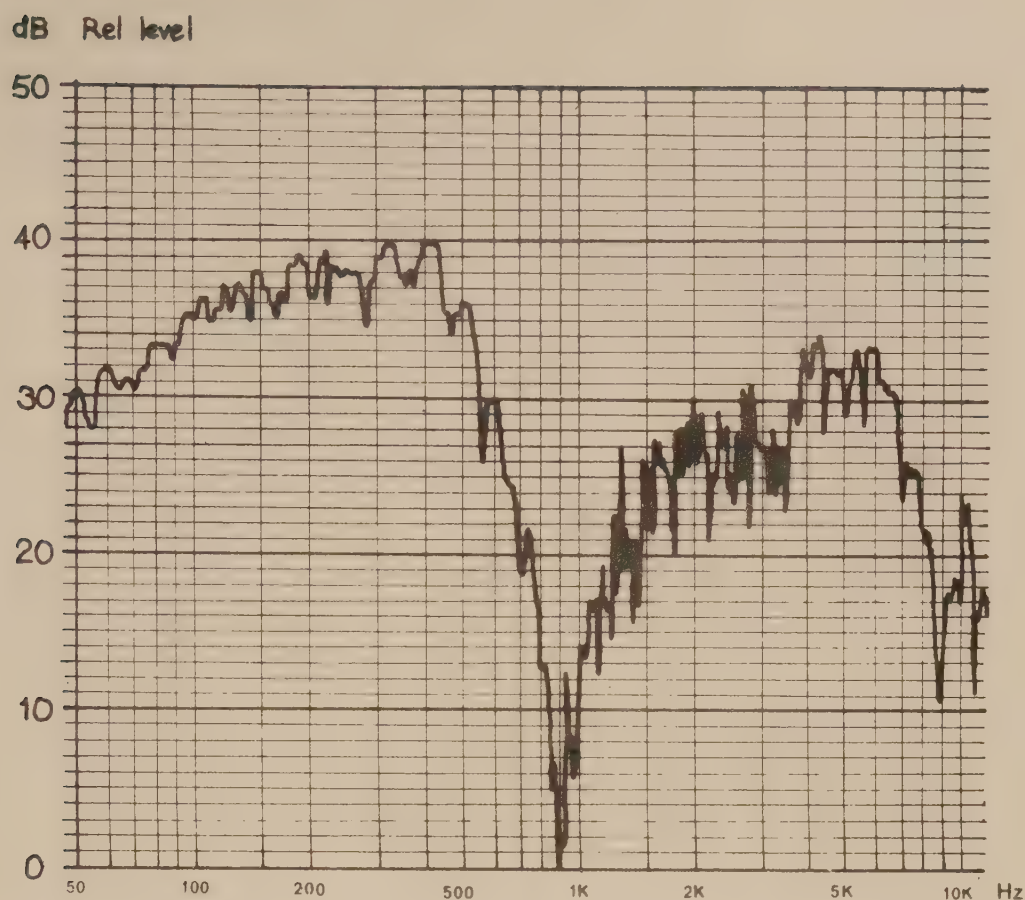


FIGURE 5.3.2. The loudspeaker and microphone elevated to 0.23 m.

The plane wave interpretation of this is that the reflexion gives a phase shift of almost π . The increment from the small distance difference between direct and reflected is added and the result is π counter-phase. The plane wave picture is not the whole truth for a point source, especially not for a source at the ground (cf. [50] and [7]). For source and receiver on the ground, the sharp dip must be explained in some other way, which both the simple model of [7] and the approximative method of [50] and [51] are able to do.

For frequencies below the extreme sound level dip we have another interesting phenomenon. At the ground the sound can be considerably stronger than higher up (in the space wave). This is very clear in the following figure from [7], showing curves obtained with the loudspeaker on the ground and the microphone in two positions, on the ground and 3.8 m above ground.

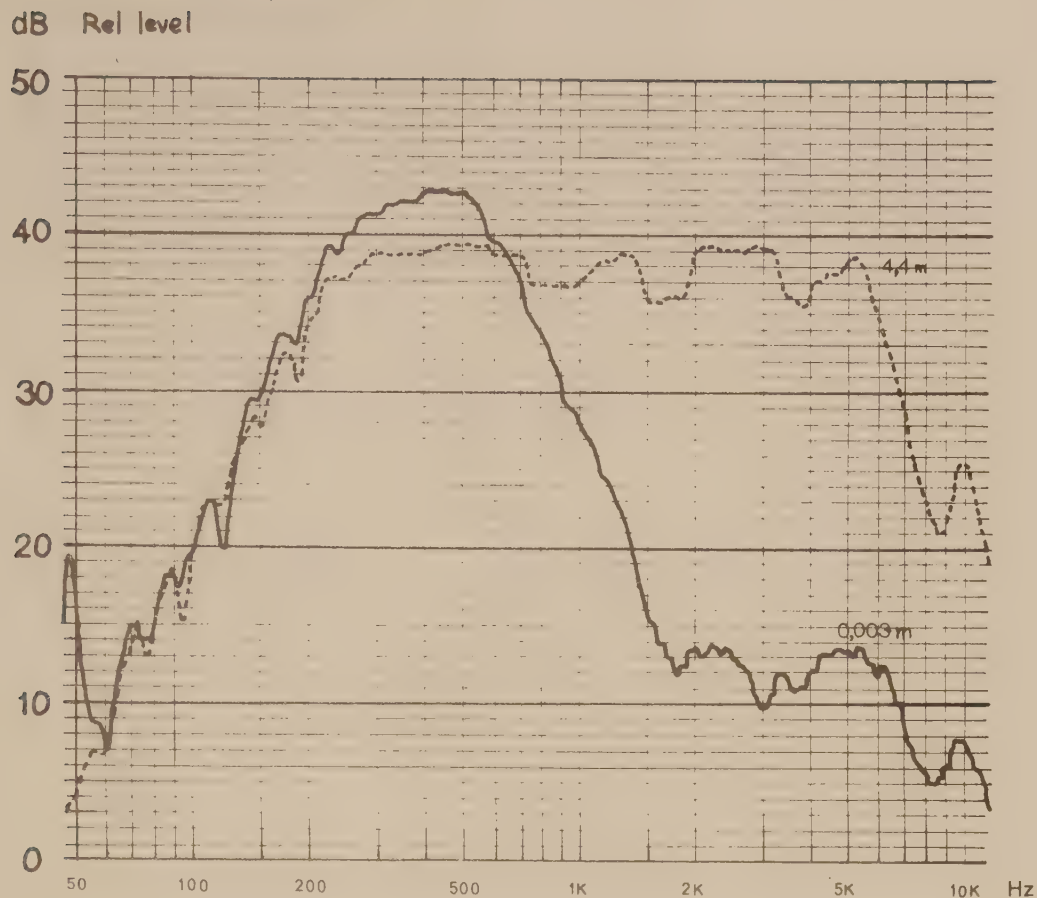


FIGURE 5.3.3. The surface wave effect.

As seen in par. 5.2 the space wave is probably almost uninfluenced by v .

The high position sound is therefore about the same as should be the case with a hard boundary. Thus the low position curve shows 3 - 4 dB more sound than a perfectly reflecting hard surface can give. Evidently this is due to a surface wave.

5.4. The surface wave

For $\xi > 1$, $jk_z = \sqrt{\xi^2 - 1} k$, so the power stays near the surface, is "ducted", in a surface wave. The power to the surface wave is obtained from the ξ -integral $(1, \infty)$. As the hard boundary has zero real part on $\xi = (1, \infty)$ we can as well use the splitted form:

$$\Pi_v = \frac{|W|^2 k^2 \rho c}{2\pi} \operatorname{Re} \int_1^{-j\infty} \frac{v}{v + \zeta} \cdot \frac{d\zeta}{j} \quad (1)$$

As mentioned we get a pole on the chosen branch and path if $\operatorname{Re} v = 0$ and $\operatorname{Im} v > 0$. The residue gives a principal value.

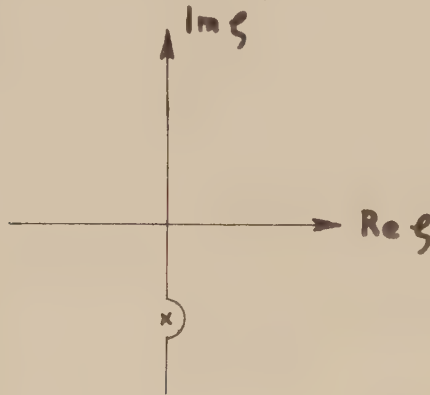


FIGURE 5.4.1. The ζ -path with $\operatorname{Re} v = 0$. Integral value = $\pi j v$.

$$\Pi_v = \frac{|W|^2 k^2 \rho c}{2} \operatorname{Im}(v) \quad (2)$$

This is $\approx \pi \operatorname{Im}(v)$ times the space wave power, $2 \operatorname{Im}(v)$ times the power of a free point source.

When $\operatorname{Re} v \neq 0$ we choose a path in the direction $(0, -v)$, as the integrand is purely imaginary on that path. The only real part, the power, is the small half-circle about the pole.

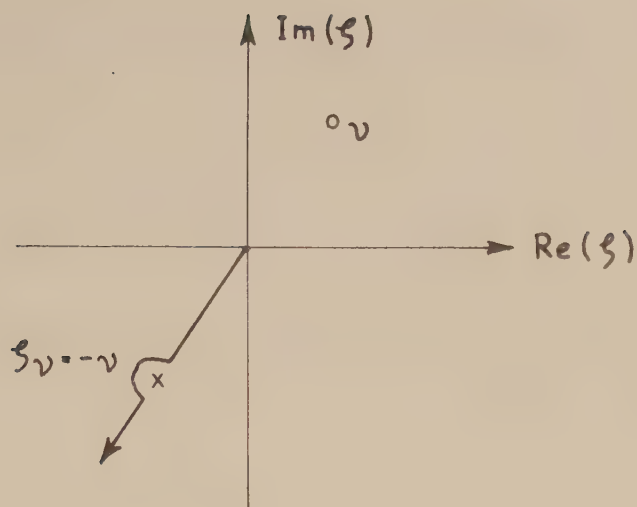


FIGURE 5.4.2. The ξ -path with $\text{Re } v > 0$.

The power is still determined by eq.(2). The physical demand of a not growing wave in the z -direction is fulfilled when $\text{Im } \xi < 0$. As $\text{Re } v \geq 0$, poles in the third quadrant from v in the first quadrant gives a surface wave. For a porous layer h , v has the form $(hr/3 + j/kh)(kh)^2$, going towards $\text{Im } v = \text{Re } v$ at larger k -values and finally towards $v = 1$.

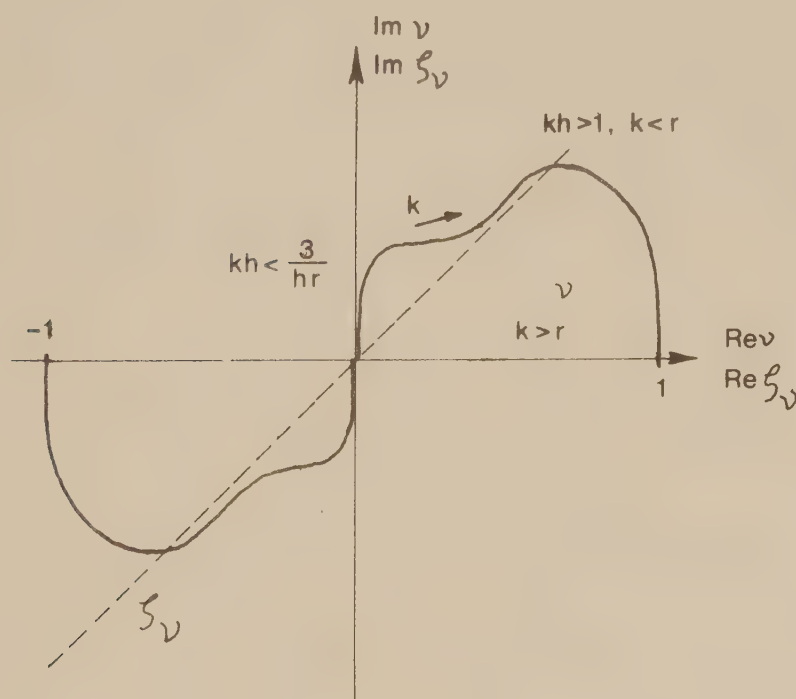


FIGURE 5.4.3. Typical situation of v for a porous layer h with normalized flow resistance r .

The wavenumber of the surface wave is obtained from the pole if $\text{Re}(\nu) \ll \text{Im}(\nu)$:

$$\kappa_\nu \approx k\xi_\nu \approx k\sqrt{1 - \zeta_\nu^2} = k\sqrt{1 + (\text{Im}(\nu))^2} \quad (3)$$

The surface wave is slower than the free wave:

$$c_\nu = \frac{c}{\sqrt{1 + (\text{Im}(\nu))^2}} \quad (4)$$

As the measure in dB is 8.7 times the measure in neper, the attenuation in z-direction is

$$-j \cdot 8.7 \cdot k\zeta_\nu = 8.7 \cdot k \cdot \text{Im}(\nu) \quad (5)$$

Normally $\text{Im}(\nu)$ first increases with increasing frequency. From a very low value at low frequency the surface wave pressure increases with the frequency as expected from par. 5.3. The geometrical attenuation is less ($1/\sqrt{x}$ compared to $1/x$) and the ν -wave is concentrated near the surface, so p_ν can be of the same order of magnitude as p_s . As f increases further the difference in wavenumbers $\approx k(\text{Im}(\nu))^2/2$ gives a counterphase state and a very low sound level.

If $0 < \text{Re}(\nu) \ll \text{Im}(\nu)$ we get the pole at ξ_ν near and below the real axis or near the $\text{Im}(\zeta)$ -axis.

$$\begin{aligned} \left(\frac{k_{\nu x}}{k} \right)^2 &= \xi_\nu^2 = 1 - \zeta_\nu^2 = 1 - (\text{Re}(\nu))^2 + (\text{Im}(\nu))^2 - j2\text{Im}(\nu) \text{Re}(\nu) \approx \\ &\approx 1 + (\text{Im}(\nu))^2 - j2\text{Im}(\nu) \text{Re}(\nu) \end{aligned} \quad (6)$$

$$\frac{k_{\nu x}}{k} \approx \sqrt{1 + (\text{Im}(\nu))^2} - j \frac{\text{Im}(\nu) \text{Re}(\nu)}{\sqrt{1 + (\text{Im}(\nu))^2}} \quad (7)$$

The complex wavenumber means attenuation along the surface:

$$8.7 \operatorname{Im}(\kappa_v) = \frac{k \operatorname{Im}(v) \operatorname{Re}(v)}{\sqrt{1 + (\operatorname{Im} v)^2}} \quad 8.7 \text{ dB/m} \quad (8)$$

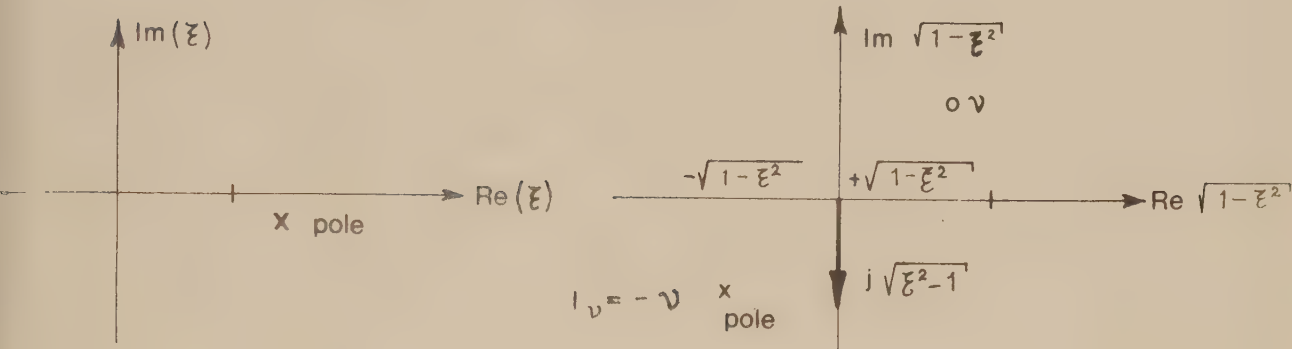


FIGURE 5.4.4. The position of ξ_v and ζ_v .

A type of surface that can give a strong surface wave is, according to eq.(2), a thin absorbent. On the ground a thin absorbent is formed by the upper layer of the soil perforated by the grass roots. From par. 3.3 we obtain for an absorbent layer h at a hard surface:

$$v \approx \left(\frac{j}{kh} + \frac{rh}{3} \right) / \left(\left(\frac{1}{kh} \right)^2 + \left(\frac{rh}{3} \right)^2 \right) \quad (9)$$

and

$$\frac{\operatorname{Re}(v)}{\operatorname{Im}(v)} = \frac{krh^2}{3} \quad (10)$$

It could be of interest to study a special case and we choose a 1 cm thick absorbent with normalized flow resistance $r = 100$ (cf. par. 3.2). At 100 Hz ($k = 1.85$) we get:

$$\frac{\operatorname{Re}(v)}{\operatorname{Im}(v)} = 6 \cdot 10^{-3} \quad (11)$$

For $v \ll 1$ and $\operatorname{Re}(v) \ll \operatorname{Im}(v)$ we look for the frequency (k -value) that

gives a maximal p_v at a distance x . The wave is attenuated by $\text{Im} \kappa_z$. We can also use the approximate formula $J_0 \approx \sqrt{1/x}$ for the geometrical attenuation.

$$p \sim \frac{\text{Im}(\nu)}{\sqrt{x}} \exp \left(- \frac{k \text{Im}(\nu) \text{Re}(\nu) x}{\sqrt{1 + (\text{Im}(\nu))^2}} \right) \approx \frac{\text{Im}(\nu)}{\sqrt{x}} \exp(-k \text{Im}(\nu) \text{Re}(\nu) \cdot x) \quad (12)$$

We use eq.(9) omitting $(rh/3)^2$ in the denominator:

$$p \sim kh \frac{1}{\sqrt{x}} \exp \left(- \frac{rh^2 k^2}{3} x \right) \quad (13)$$

$$\frac{dp}{dk} \sim 1 - 2 \frac{k^2 rh^2}{3} x = 0 \quad (14)$$

$$k_1 = \sqrt{\frac{3}{2rx}} \cdot \frac{1}{h}$$

$$\text{or} \quad (15)$$

$$f_1 \approx 70 \frac{1}{h} \cdot \frac{1}{\sqrt{rx}}$$

With the surface example leading to eq.(11) we get:

$$k \approx 12/\sqrt{x} \quad (16)$$

or

$$f = 680/\sqrt{x} \quad (17)$$

This is a typical value for grass (cf. par. 5.3).

5.5. A plane transform method and Sommerfeldt's angular spectrum method

In [48] Sommerfeldt gave a solution for radiowaves above a conducting plane. The transform he used was given by Cauchy and is not a k -transform, as $k^2 |\gamma_k|^2 = k_x^2 + k_y^2 + k_z^2 = k^2$ is constant during the transform integral. We shall refer to the method as an angular transform and an angular spectrum. For a point source we obtain:

$$\frac{4\pi p}{Wk^2 \rho c} = \frac{4\pi \psi}{-jkW} = F = \frac{e^{-jkr}}{jkr} \quad (1)$$

This is expanded as an integral over k -directions:

$$F = \frac{1}{2\pi} \int_L \int_0^{2\pi} e^{-jk \vec{\gamma}_k \cdot \vec{r}} d\vec{\gamma} \quad (2)$$

L is an integration path, which shall be determined later.

The polar coordinates to the γ_z -axis (transform axis) are:

$$\begin{aligned} \gamma_{kx} &= \sin \beta \cos \alpha \\ \gamma_{ky} &= \sin \beta \sin \alpha \\ \gamma_{kz} &= \cos \beta \end{aligned} \quad (3)$$

A point on the z -axis gives a simple expression in polar coordinates to the x -axis:

$$F = \frac{1}{2\pi} \int_L \int_0^{2\pi} e^{-jkz \cos \beta} \sin \beta \, d\alpha d\beta = \int_L e^{-jkz \cos \beta} \sin \beta \, d\beta \quad (4)$$

If the integration path L is real and $(0, 2\pi)$, as should be the case for an isotropic transform with respect to direction in the space, we get a standing wave solution:

$$F = \frac{\sin(kz)}{kz} \quad (5)$$

In standard works [8] a complex path is introduced $(0, \pi/2, j\infty)$, which replaces $(0, 2\pi)$ and gives the wanted outgoing wave. The disadvantage with such a deduction is that the unexperienced reader might get the expression that the transform is still isotropic and general with respect to direction. Sommerfeldt also used the transform in a different form and we shall find an integral on the real axis leading to the mentioned β -path.

We shall try to improve the physical picture and revert to the method in par. 5.2. We transform the point source into a source layer in the x -, y -plane, κ -plane, with δ -thickness:

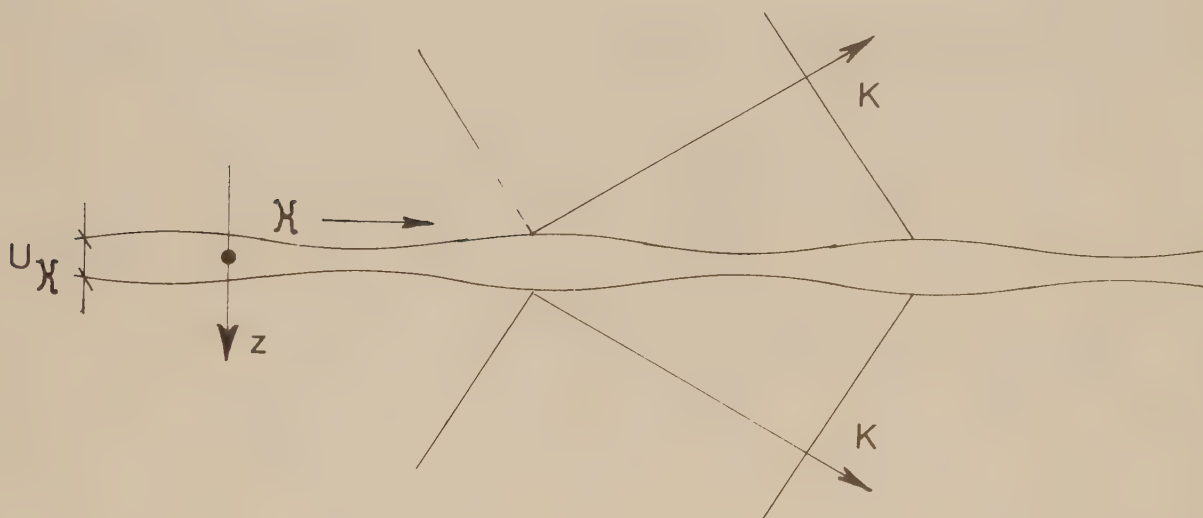


FIGURE 5.5.1. The transform source layer.

The transform is the same as for a point source on a hard plane. The source field power is only the half, as the radiation resistances are parallell. There are two waves, one for $z > 0$ and one for $z < 0$, that divide the power of the u_{κ} -component. The halving of power and division in two waves is obvious from the following network:

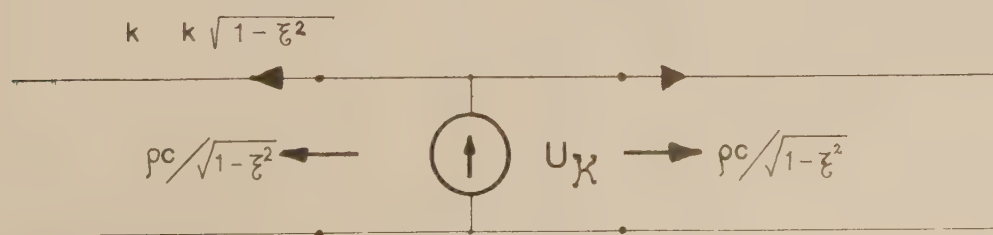


FIGURE 5.5.2. The u_{κ} -radiation.

Hence the pressure component is

$$p_k = u_k \cdot \frac{1}{2} \rho c / \sqrt{1 - \xi^2} \quad (6)$$

The load is the same on both sides and the velocity is divided in two equal parts. The field is the same on both sides (denoted $|z|$). We get the normalized pressure $F = 4\pi p / (\rho c k^2 W)$:

$$F = \frac{1}{2\pi} \int_0^{2\pi} \int_0^\infty \frac{\exp(-jk(|z| \sqrt{1 - \xi^2} + \xi(x \cos \alpha + y \sin \beta)))}{\sqrt{1 - \xi^2}} \xi d\xi \quad (7)$$

We can solve the integral and for simplicity we obtain on the z -axis:

$$F = \int_0^\infty \frac{\exp(-jk|z| \sqrt{1 - \xi^2})}{\sqrt{1 - \xi^2}} \xi d\xi = \frac{e^{-jk|z|}}{jk|z|} \quad (8)$$

If we substitute with $\xi = \sin \beta$ we arrive at the angular transform:

$$\int \exp(-jk|z| \cos \beta) \cdot \sin \beta d\beta \quad (9)$$

The ξ -integral directly gives $F = \exp(-jk|z|) / (jk|z|)$ as expected. The β -integral gives of course the same value if $L = \beta_\xi$, where β_ξ is directly given by the substitution

$$\begin{aligned} \xi \leq 1: \quad \beta_\xi &= \arcsin \xi \\ \xi \geq 1: \quad \beta_\xi &= \pi/2 + j \ln(\xi + \sqrt{\xi^2 - 1}) \end{aligned} \quad (10)$$

The mapping of the positive real ξ -axis on the β -plane, the β_ξ -path, is shown in FIG 5.5.3.

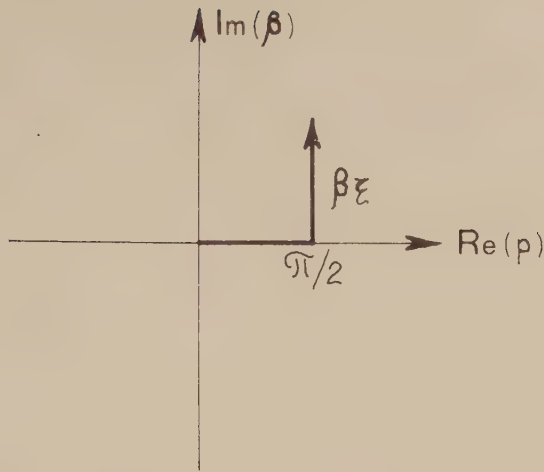


FIGURE 5.5.3. The β_ξ -path.

By this deduction of (9) and (10) we realize that, as (8) is specialized on the (x, y) -plane, also (9) is specialized on the z -direction. This is of great importance when a surface boundary is added.

Often a new "steepest descent" path is introduced. According to Cauchy this gives the same integral value if there are no singularities between the original path and the new path.

The new path β_t is obtained from the following connection, where t is real:

$$\cos(\beta_t) = 1 - jt \quad (11)$$

Inserting this in (9) leads to:

$$F = e^{-jkz} \int_0^\infty e^{-kzt} \cdot \frac{dt}{j} = \frac{e^{-jkz}}{jkz} \quad (12)$$

The integrand monotonously decreases, ξ_t is a steepest descent path.

The corresponding path β_t in the β -plane is obtained from (11):

$$\beta = \frac{1}{j} \ln \left(1 - jt \pm \sqrt{-t^2 + 2jt} \right) \quad (13)$$

The β_t -path is shown in the following β -plane:

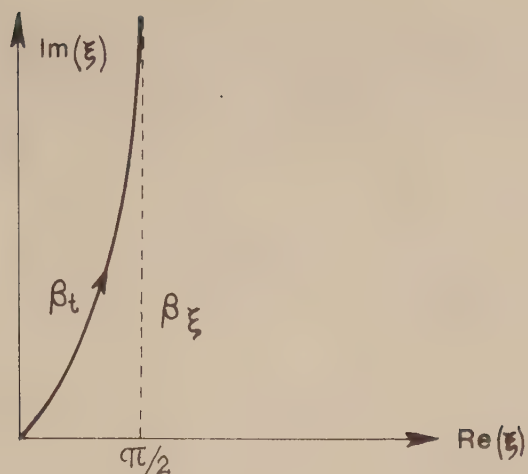


FIGURE 5.5.4. The β_t -path in the β -plane.

If instead the t -substitution is made in (8) we naturally get the same result ($\sqrt{1 - \xi^2} = 1 - jt$):

$$\xi_t = \sqrt{t^2 + 2jt} \quad (14)$$

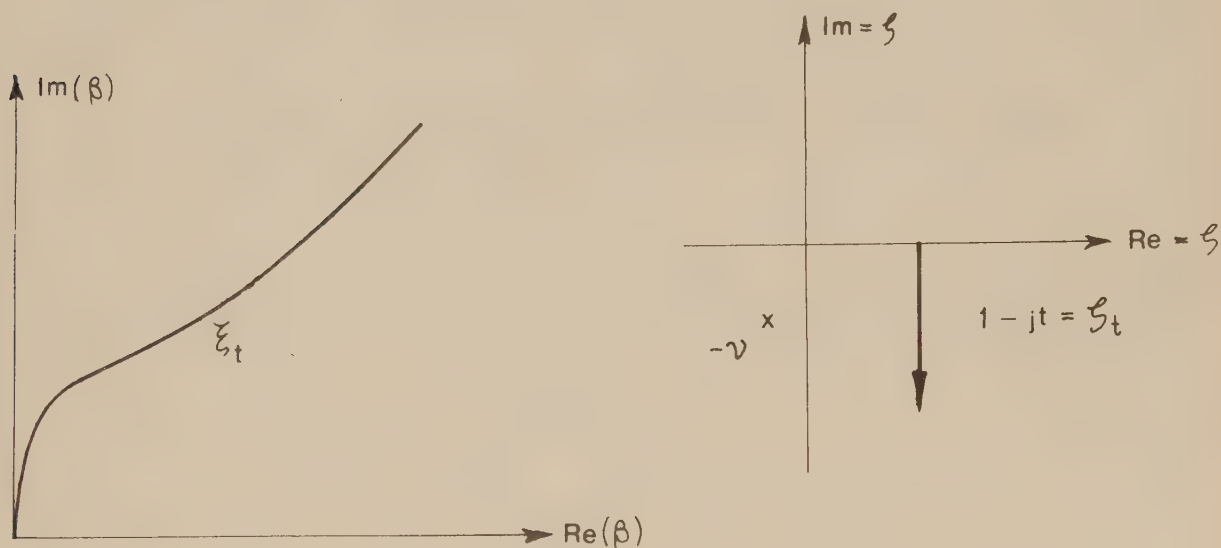


FIGURE 5.5.5. The ξ_t -path in the ξ -plane and $\sqrt{1 - \xi_t^2}$ -path = ζ_t -path.

A dipole source can be treated very easily by the layer transform:

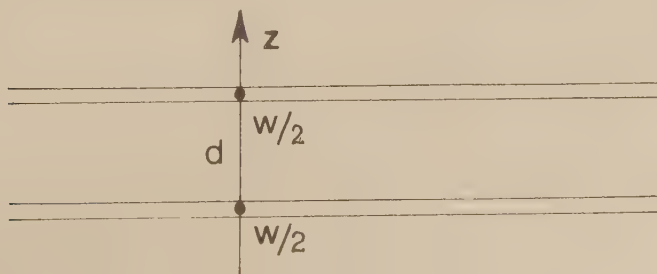


FIGURE 5.5.6. The dipole transform planes.

The pressure in one of the point sources and $W/2$ gives half the power. We get the pressure from both sources at each source, which directly gives:

$$\Pi_d = \text{Re} \frac{|W|^2 k^2 \rho c}{4\pi} \cdot \frac{1}{4} \int_0^\infty \frac{1}{\sqrt{1 - \xi^2}} \left(1 - e^{-jkd\sqrt{1 - \xi^2}} \right) \xi d\xi \quad (15)$$

With $d \ll \lambda$ we can use the first three terms of the expansion for the exponential. To shorten the expression we introduce the power Π_p for a point source with the volume velocity W :

$$\Pi_d = \Pi_p \cdot \frac{1}{2} \text{Re} \int_0^\infty \frac{1 - 1 + jkd\sqrt{1 - \xi^2} + \frac{1}{2}(kd\sqrt{1 - \xi^2})^2}{\sqrt{1 - \xi^2}} \xi d\xi \quad (16)$$

The third term of the expansion is real, but only on $(0, 1)$, so we get:

$$\Pi_d = \Pi_p \frac{(kd)^2}{4} \int_0^1 \sqrt{1 - \xi^2} \xi d\xi = \Pi_p \frac{(kd)^2}{4} \int_0^1 \zeta^2 d\zeta = \Pi_p \frac{(kd)^2}{12} \quad (17)$$

The transform is as usual easier to handle in power and radiation resistance calculations than in integration of intensity about the source.

5.6. The point source above a v -plane

We shall now go over to a central task of this part, which is to study the point source above a locally reacting plane surface.

From radiation problems we know that the boundary condition can be fulfilled by simple expressions. The wave component and the surface must have the same properties in all points. Therefore we use the plane transform, which directly gives the wanted simple form of the boundary condition. The transform plane has to be parallel to the v -plane. If this is not the case, the infinite reflexion factor demanding that the point reflexion is typical for the whole plane, cannot be used. All sources but a plane source layer must lead to integral equations, e.g. using Green's formulas.

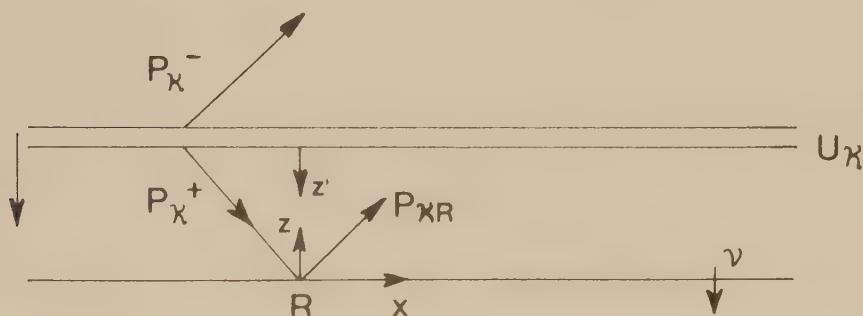


FIGURE 5.6.1. The transform layer.

The transform components have $k'_z = k\sqrt{1 - \xi^2}$, so the boundary condition is as usual fulfilled with

$$(p_k^+ + p_{kR}) v = u_z = (p_k^+ - p_{kR}) k_z / k \quad (1)$$

$$R = \frac{p_{kR}}{p_k^+} = \frac{k_z - kv}{k_z + kv} = 1 - \frac{2}{1 + \sqrt{1 - \xi^2} / v} \quad (2)$$

The total pressure is obtained as an integral of the component pressure:

$$p = \frac{Wk^2 \rho c}{(2\pi)^2} \int_0^{2\pi} \int_0^\infty \frac{\exp(-jk|z-h|\sqrt{1-\xi^2}) + R \exp(-jk|z+h|\sqrt{1-\xi^2})}{2\sqrt{1-\xi^2}} \cdot \exp(-jk\xi(x \cos \alpha + y \sin \alpha)) \cdot \xi d\xi d\alpha \quad (3)$$

As $R = 1 - 2v/(v + \sqrt{1-\xi^2})$ it is convenient to divide the reflected sound in a hard boundary part and a v -part from the two terms of R .

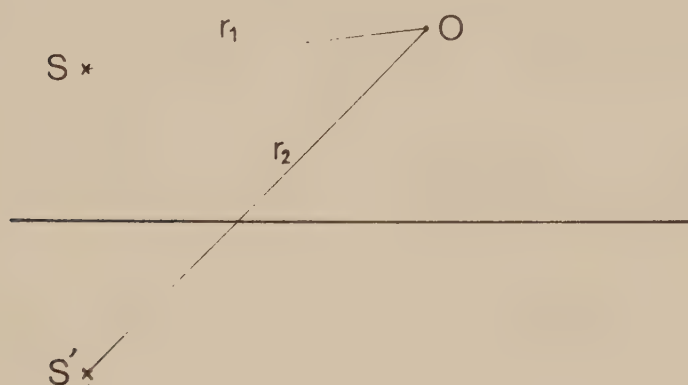


FIGURE 5.6.2. Sources and observation point.

The term "1" is the hard boundary reflexion and we obtain:

$$p = \frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^\infty (\{p_k^+ \text{ or } p_k^-\} + p_{kR}) k dk d\alpha =$$

$$= p_{\text{direct}} + p_{\text{hard}} - \frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^\infty p_{kv} k dk d\alpha \quad (4)$$

The last term is only produced if $v \neq 0$ and will be referred to as Q . For the field of normalized pressure we get:

$$\frac{4\pi p}{Wk^2 \rho c} = F = F_d + F_h - Q \quad (5)$$

Without loss of generality we put $y = 0$. We obtain Q :

$$Q = \frac{1}{2\pi} \int_0^{2\pi} \int_0^\infty \frac{2v \exp(-jk((z+h)\sqrt{1-\xi^2} + \xi x \cos \alpha))}{\left(v + \sqrt{1-\xi^2}\right) \sqrt{1-\xi^2}} \xi d\xi \quad (6)$$

If the deduction is made in the velocity potential ψ we get the same field with F according to

$$F = \frac{4\pi p}{jkW} \quad (7)$$

If the source is at the surface, $h = 0+$, $z = 0+$, we get directly from (5):

$$\frac{1}{2\sqrt{1-\xi^2}} \left(1 + 1 - \frac{2v}{v + \sqrt{1-\xi^2}} \right) = \frac{1}{v + \sqrt{1-\xi^2}} \quad (8)$$

This is evidently the same result as was directly obtained in par. 5.2, using the radiation method. Obviously the transform source layer can be put on a surface or in space. The important demand is however that R shall be valid for the transform field.

We also observe that $v = -\sqrt{1-\xi^2}$ at the surface pole. The surface wave in Q is exactly the same for $h = 0+$ as the one obtained in par. 5.2 for the complete field.

We perform the α -integral (cf. par. 5.2) and in the x -, z -plane we get:

$$Q = \int \frac{2v \exp(-jk(z+h)\sqrt{1-\xi^2})}{\left(v + \sqrt{1-\xi^2}\right) \sqrt{1-\xi^2}} J_0(kx\xi) \xi d\xi \quad (9)$$

The Q -term will produce the surface wave component of the field. For $\xi > 1$ the factor $\exp(-jkh j\text{Im}\sqrt{1-\xi^2})$ is an attenuating factor $\xi_v > 1$, so the surface wave is multiplied by

$$\exp\left(+\text{Im}\sqrt{\xi_v^2 - 1}\right) = \exp(\text{Im}\zeta_v) = \exp(-kh \text{Im}v) \quad (10)$$

As expected a weaker surface wave is produced when the source is elevated.

For small v -values (grass ground) a further property of the field is that Q_S , the integral (0, 1) is very small compared to $F_d + F_h$. Thus the observed interference in par. 2.3 is normally between $(F_d + F_h)$ and Q , the surface wave.

For elevated source and observer usual reflexion interference will of course occur. However, the level dips are narrower when produced by F_d and F_h than by $(F_d + F_h)$ and Q .

If we insert $\xi = \sin \beta$ we obtain:

$$Q = \int_{L=\beta_\xi} \frac{2ve^{-jk(z+h)\cos \beta}}{v + \cos \beta} J_0(kx \sin \beta) \sin \beta d\beta \quad (11)$$

This is equivalent to the layer transform, so the transform axis (z -axis) must be normal to the surface. This very clear demand from our deduction shall be further discussed. In Weyl's method the transform axes are tilted and the κ -transform layer cuts into the v -plane.

5.7. Weyl-Ingård's angular method above a plane

In this paragraph I shall present a solution that only seems to be approximately right for part of the transform range. We have seen that the boundary conditions only are fulfilled for a transform plane parallell to the v -plane. However, Weyl's solution below is related as if the reflexion used was permitted.

A method that has been frequently used in acoustics originates from Weyl [49], who argued that Sommerfeldt's method with the polar axis perpendicular to the surface was "das Problem zu wenig angepasst", not adapted to the problem. Weyl instead used the directions source-observer, SO , and image-source-receiver, $S'O$, as polar axes for the angular transform. For a locally reacting surface Ingård [50] has developped Weyl's method according to the following. This method was also adopted by Jonasson [51], who however mentioned discrepancies for low frequencies near the ground.

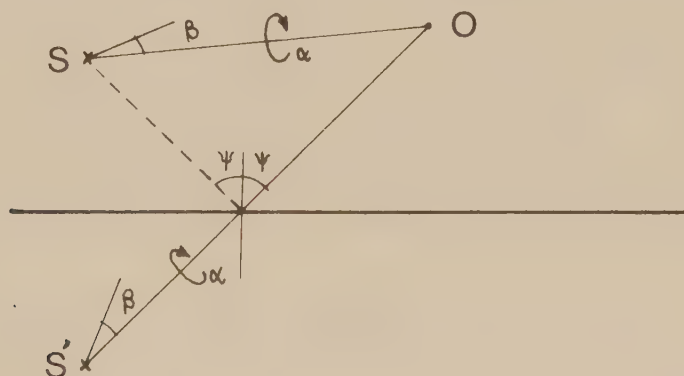


FIGURE 5.7.1. Weyl's transform axes.

The reason for choosing the transform directions shown in the figure is that the transform is easiest to invert on the transform axis, which we realize from par 5.2 and 5.5.

The advantage of the chosen polar axes is thus that the α -integration does not involve the exponential function. This only depends on β , $\exp(-jkr \cos \beta)$. The reflexion factor R is more complicated than in Sommerfeldt's method and is involved in the α -integration. ϕ is not equal to β . From figure 5.7.1. we obtain:

$$\cos \phi = \sin \psi \cdot \sin \beta \cdot \cos \alpha + \cos \psi \cdot \cos \beta \quad (1)$$

$$R(\beta, \alpha) = 1 - \frac{2v}{v + \cos(\phi(\beta, \alpha))} \quad (2)$$

ϕ is the angle between the transform wave component and the surface normal.

As before we use Q for the field produced by v :

$$Q = \int_0^\pi \int_L \frac{2v \cdot \exp(-jkr_2 \cos(\beta))}{v + \cos(\phi(\beta, \alpha))} \quad (3)$$

The integrand involving α can be rewritten as:

$$\frac{B(\beta)}{1 + A(\beta) \cdot \cos \alpha} \quad (4)$$

This is a known integral on α given in standard tables. Performing the α -integral we obtain:

$$Q = \int_L \frac{2v \cdot \exp(-jkr_2 \cos \beta) \sin \beta \cdot d\beta}{((v + \cos \psi \cos \beta)^2 - (\sin \psi \cdot \sin \beta)^2)^{1/2}} \quad (5)$$

According to [50] and [51] the integral path is chosen in the same way as for a free field. With $\cos \beta = 1 - jt$ the integration is $(0, \infty)$:

$$Q = \int_0^\infty \frac{2v \cdot \exp(-jkr_2(1 - jt))}{((v + \cos \psi(1 - jt))^2 - (\sin \psi(t^2 + 2jt))^2)^{1/2}} \cdot \frac{dt}{j} \quad (6)$$

The surface wave seems to be more concealed in this form of solution. Weyl also argued that there was no such wave and it seems not to be shown in Ingård's solution. Jonasson [51] also reports discrepancies in the range, where I consider the surface wave to be important.

We shall compare the different methods further and first we choose the grazing case with $h = 0$ and $z = 0$. We adopt the original real axis integral also to Ingård's solution, to avoid all problems with branch cuts and poles.

(Q_W , Weyl-Ingård; Q_S , Sommerfeldt.)

$$Q_W = \int_0^{\infty} \frac{2v \exp(-jkx\sqrt{1-\xi^2})}{\sqrt{v^2-\xi^2} \cdot \sqrt{1-\xi^2}} \xi d\xi \quad (7)$$

$$Q_S = \int_0^{\infty} \frac{2v J_0(kx\xi)}{(v + \sqrt{1-\xi^2})\sqrt{1-\xi^2}} \xi d\xi \quad (8)$$

(8) will give the same surface wave as we obtained in par. 5.2. The power was obtained from the pressure at the source point:

$$Q_{W0} = \int_0^{\infty} \frac{2v}{\sqrt{v^2-\xi^2}} \cdot \frac{\xi d\xi}{\sqrt{1-\xi^2}} d\xi \quad (9)$$

For comparison we also look at:

$$Q_{S0} = \int_0^{\infty} \frac{2v}{(v + \sqrt{1-\xi^2})\sqrt{1-\xi^2}} \xi d\xi \quad (10)$$

In par. 5.2 and 5.4 we have studied the surface wave for $0 < \text{Im}v \ll 1$ and $\text{Re}v \ll \text{Im}v$. In (9) it is not evident as for (10) by the factor $\exp(-zk\sqrt{\xi^2-1})$, that $\xi > v$ gives a surface wave. In (5) we can study the rapid field decrease near $z = 0$. For small z -values (the height of 0) the denominator of (5) is:

$$\approx ((v + (z/x)\sqrt{1-\xi^2})^2 - \xi^2)^{1/2} \quad (11)$$

At $v = \xi$ there is a singularity in (9). (11) is zero for $z = 0$. There is a rapid attenuation for increasing z , so $\xi \approx v$ could give a surface wave increment to the integral (9).

This is also natural as ξ stands for the perpendicular part of wavenumber

$(\sin \beta)$ in (9) and $\sqrt{1 - \xi^2}$ is this part in (10) $(\cos \beta)$. $\xi = \nu$ is no pole, but a branch point, so the evaluation is not as self-evident as for (10).

The normalized power into the surface wave according to (10) is obtained from eq.(5.4.2):

$$2\pi \operatorname{Im}(\nu) \quad (12)$$

It is an imaginary ν that gives a strong surface wave. $((\operatorname{Im}\nu)^2 + \xi^2)$ is never zero on the real axis. The surface wave cannot be isolated in the solution as in par. 5.2 and 5.6.

If the source is on the ground and the observation point above we get $\cos \gamma = 0$ in (5) and furthermore, with $\cos \beta = \sqrt{1 - \xi^2}$ we obtain:

$$Q_W = \int_0^\infty \frac{2\nu \cdot \exp(-jkz\sqrt{1 - \xi^2})}{\sqrt{(\nu + \sqrt{1 - \xi^2})^2} \cdot \sqrt{1 - \xi^2}} \xi d\xi \quad (13)$$

As expected $Q_W = Q_S$ with identical transform planes, as is the case with 0 above S.

In [59] some different solutions are discussed. It is pointed out that the solution form depends on the choice of transform axes.

There are many other papers on the evaluation of the integrals according to Weyl. It seems as if the problems with the even more difficult evaluation of the original Sommerfeldt solution have led to discussions of different methods of evaluating Weyl's solution [59]. However, comparisons to Sommerfeldt's transform [59] are seldom made.

5.8. Transform planes and boundaries

The only transform plane or real radiating surface that directly gives a known k_2 identical in all points of an infinite (x, y) -plane (v -plane) is the infinite plane parallel to the v -plane. This type of plane was used in par. 5.6 but not in par. 5.7, where the transform plane had directions according to the following figure.

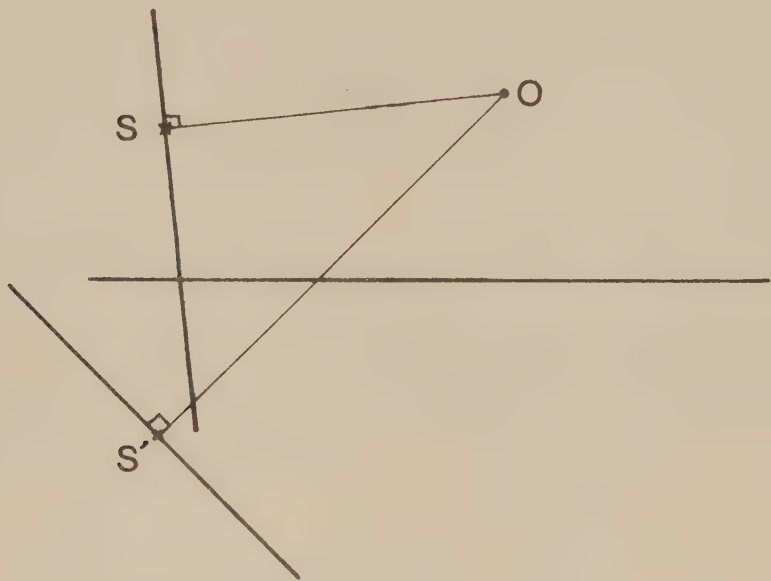


FIGURE 5.8.1. Transform planes.

The "plane" waves from the vertical source planes are approximately correctly reflexed except at the cut between the planes, where edge effects must occur. This is probably the reason why Ingård's solution [50] as shown by Jonasson [51] can give acceptable results for the frequency range, where $\xi < 1$ and the surface wave is less important.

For $\xi > 1$ the vertical transform planes produce a nearfield that cannot even approximately be correctly dealt with without an integral equation. Hence the surface wave is not correct and the results are wrong at the surface.

Another fact is that the upward components are reflected. For the grazing case we have $\cos \alpha > 0$ for $-\pi/2 < \alpha < \pi/2$ and $\cos \alpha < 0$ for $\pi/2 < \alpha < 3\pi/2$. The v -part of the reflexion factor is:

$$\frac{2v}{v + \sin \beta \cos \alpha}$$

(1)' "

It is difficult to judge about the effect of this, but the sign shift of $\cos \alpha$ is remarkable.

For the grazing case the transform plane is vertical in the Q_W -solution.

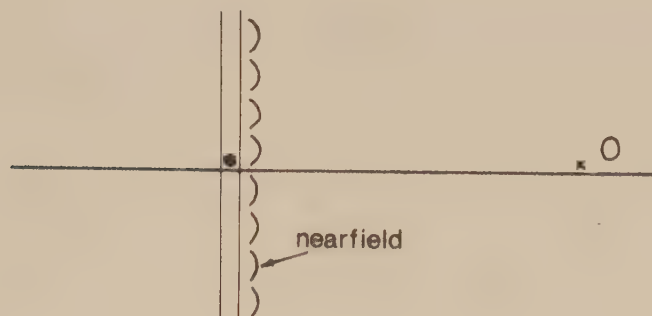


FIGURE 5.8.2. Grazing Q_W -plane.

The plane waves for $\xi < 1$ are still reflected fairly correct. The plane cannot radiate waves with $\kappa_x > k$ ($\sqrt{1 - \xi^2} > 1$) along the boundary like the Sommerfeldt situation.

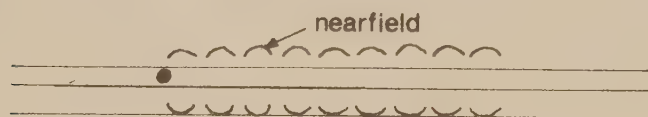


FIGURE 5.8.3. $k\xi = k\xi_v$ gives the surface wave.

It seems as if one expression for the missing or not sufficiently strong surface wave is that the vertical plane does not produce waves along the boundary v that can coincide with the v -wave, which has $\kappa_v > k$. It is only a nearfield in the cut between the planes, that acts on the v -plane.

The vertical transform plane does not fulfil the boundary condition strictly. In such a case an integral equation can be the solution, e.g. from Green's formula and the wave equation.

If the boundaries are not simple and permit direct adding of solutions to the differential equation, e.g. by the reflexion factor, some integral form must be used. With two-potential fields A and B , for a volume V with

the enclosed surface S we get according to Green:

$$\iiint_V (A\Delta B - B\Delta A) dV = \iint_S \left(A \frac{\partial B}{\partial n} - B \frac{\partial A}{\partial n} \right) dS \quad (2)$$

With the volume cleared from singularities A and B can fulfil the homogeneous wave equation:

$$\begin{aligned} \Delta A &= k^2 A \\ \Delta B &= k^2 B \end{aligned} \quad (3)$$

Then the left member in eq.(1) is zero. We choose B as a Green's function $(1/(4\pi r)) \cdot \exp(-jkr)$, a "spherical wave" from the observation point. With the centre of B in the observation point we get a small spherical surface S_2 just about O , where B dominates the derivation:

$$\iint_S -A \frac{\partial}{\partial r} \frac{e^{-jkr}}{r} = A \cdot 4\pi r^2 \frac{(jk + 1/r)}{4\pi r} e^{-jkr} \xrightarrow{r \rightarrow 0} A(O) \quad (4)$$

$$A(O) = \frac{1}{4\pi} \iint_{S_1} \left(A \frac{\partial e^{-jkr}}{\partial r \cdot r} - \frac{e^{-jkr}}{r} \cdot \frac{\partial A}{\partial n} \right) dS \quad (5)$$

This is an integral solving A in implicit form. The idea is that A can often be easier approximated in S_1 than in O . An application on sound field over grass land is given in [7].

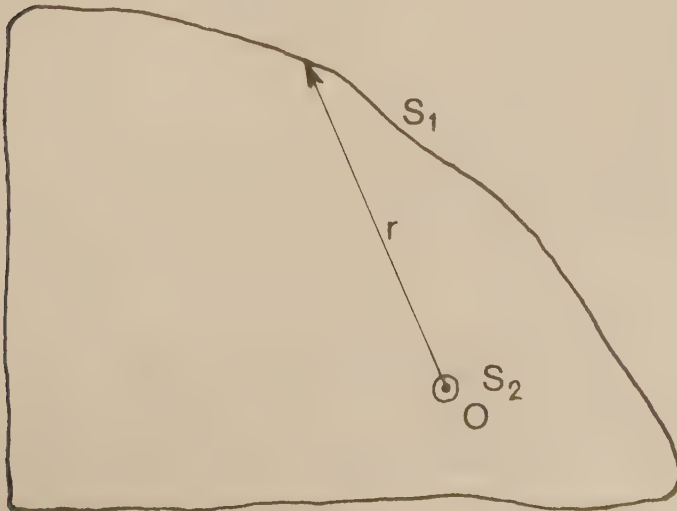


FIGURE 5.8.4. The integration surface.

5.9. Comparison between approximative methods also involving sound barriers

Jonasson [51] and I myself [7] have dealt with the problem of insertion loss for sound barriers on the ground. The reason for bringing up the problem was that the insertion loss for a sound barrier in practice turned out to be smaller on the ground than on a hard surface. This is mainly due to a strong ground effect, leading to high attenuation. The effect is stronger without than with the barrier, so the insertion loss is decreased.

In spite of the missing surface wave and non-perfect boundary fulfilling properties, Ingård's solution has proved to be useful in practice. This is also shown by Jonasson in [51].

The reason why measured values and Weyl-Ingård's theory can agree, is that the Weyl transform still avoids to depend only on the plane wave reflexion factor R_o :

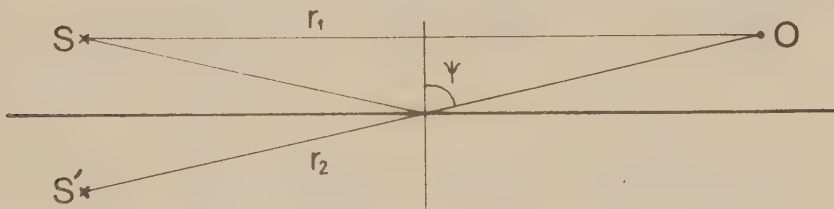


FIGURE 5.9.1. Plane wave reflexion factor.

$$R_o = 1 - \frac{2v}{v + \cos \psi} \quad \quad \quad (1)$$

$$p = p_{\text{direct}} \left(1 + R_o e^{-jk(r_2 - r_1)} \right) \xrightarrow{\psi \rightarrow \pi/2} 0$$

For very low positions $R_o \rightarrow 0$ for a porous surface, so the integration over other angles of incidence always gives a better result than just the plane wave reflexion.

There are other approximative methods to avoid R_o and get other angles of incidence. In [7] I used Green's formulas of Huygens's principle according to the following simple approach:

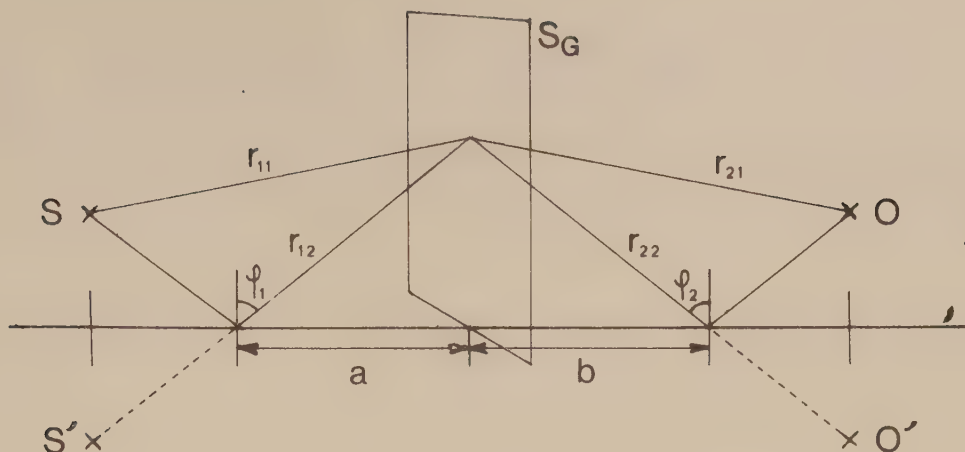


FIGURE 5.9.2. The integration plane.

Instead of taking the sound directly I integrated over a surface between source and receiver:

$$\Delta r_1 = r_{12} - r_{11}: \quad A \approx \left(1 + R(\phi_1) e^{-jk\Delta r_1} \right) \frac{e^{-jkr_{11}}}{r_{11}} \quad \} \quad (2)$$

$$\Delta r_2 = r_{22} - r_{21}: \quad B \approx \left(1 + R(\phi_2) e^{-jk\Delta r_2} \right) \frac{e^{-jkr_{21}}}{r_{21}}$$

Now we assume that A on S_G is the sound from the point source in S and the reflected sound. Also the Green's function from O is reflected. We get the solution according to eq.(5.8.12).

This method functions very well also for source and receiver on the ground, which gives $R_0 = -1$ and $F_0 = 0$, if the plane wave reflexion is used.

The final goal was to get a proper insertion loss for a sound barrier. This was placed in the integration surface and the reduction of area in the integral showed the insertion loss.

The integration area can be regarded as a Green area for integration or a Huygens area for elementary sources. The method gave very good agreement between measurement and practical insertion loss. With a barrier the

plane wave R_0 always gives a rather true result, but without a barrier the computation is more critical. Agreement in insertion loss is therefore a sign of a good method without the sound barrier. We used $R_0(\phi)$, but the integration minimized the error to the lower part of the integration surface.

Examples from [7] are shown below, both for attenuation and insertion loss.

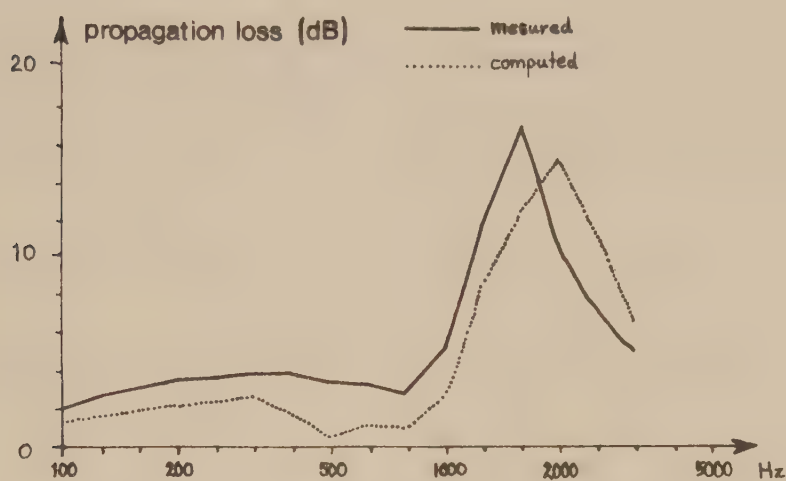


FIGURE 5.9.3. Propagation loss.

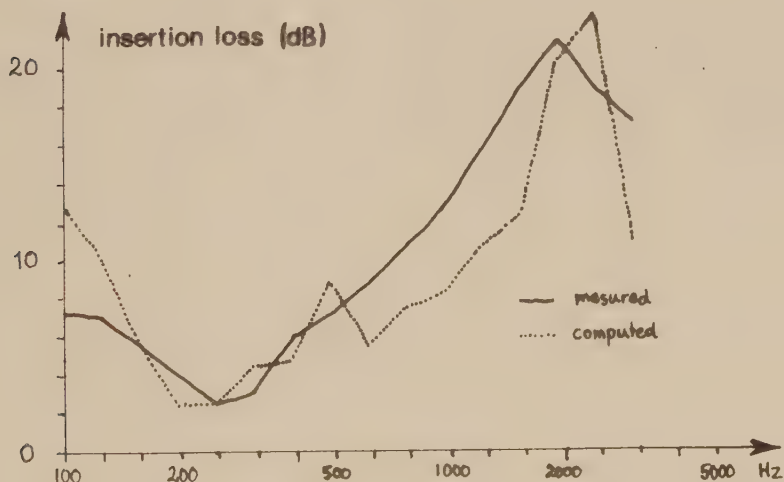


FIGURE 5.9.4. Insertion loss.

This simple form of solution shows good agreement and uses real waves but could not give a surface wave.

The same is probably valid for the solutions used in [51]. The solutions in [50] and [51] are not exact at the boundary and do not give the surface wave. They are useful approximations just like the simple model in [7]...

5.10. Transform cases

We have seen that the two-dimensional layer transform is well adapted to the problem of a plane surface. Just to get a complete picture of the transforms, for a while we shall revert to the three-dimensional κ -transform. The purpose of this paragraph is to give a physical interpretation of the three-dimensional κ -transform. This leads to plane source waves given by the three transform κ -numbers.

In a two-dimensional transform we have earlier obtained $u_\kappa = W \text{ m}^3/\text{s}$ or $u \text{ m/s per (wavenumber)}^2$. Analogous we shall now get $\phi_\kappa = W$ but interpret this as $\phi_\kappa \text{ s}^{-1} \text{ per (wavenumber)}^3$. It will be shown by the following:

$$\text{Transform wavenumber: } \vec{\kappa} = (\kappa_x, \kappa_y, \kappa_z) \quad (1)$$

We get a "white" distribution from the three-dimensional transform:

$$\phi_\kappa = \iiint e^{+j\vec{r}\vec{\kappa}} \cdot W\delta(\vec{r}) d\vec{r} = W \quad (2)$$

The ϕ_κ -component is a distribution of sources similar to a plane wave. The connected pressure and particle velocity can be calculated from the plane continuity and Euler equations. The source is added in the continuity equation. This can be compared with the one-dimensional interspace wave in par. 3.7.

With the notations in Part 3, $z_1 = j\omega\rho$ and $y_c = jk/\rho c$, we add the source in the continuity equation:

$$z_1 u_\kappa = -j\kappa p_\kappa \quad (3)$$

$$y_c p_\kappa = -j\kappa u_\kappa + \phi_\kappa \quad (4)$$

We solve the corresponding pressure

$$\frac{p_\kappa}{\phi_\kappa} = \frac{1}{y_c(1 - (\kappa/k)^2)} = \frac{\rho c/(jk)}{1 - (\kappa/k)^2} \quad (5)$$

and the corresponding velocity

$$\frac{u_{\kappa}}{\phi_{\kappa}} = \frac{j/k}{1 - (\kappa/k)^2} \quad (6)$$

The power is obtained from the source point pressure $\iiint p_{\kappa} d\kappa_x d\kappa_y d\kappa_z$. In polar κ the integral element is $4\pi\kappa^2 d\kappa = 4\pi k^3 \xi^2 d\xi$. $\xi = \kappa/k$ and we get:

$$p_0 = \frac{W}{(2\pi)^3} \int_0^{\infty} \frac{4\pi k^2 \rho c / j}{1 - \xi^2} \xi^2 d\xi \quad (7)$$

$$\Pi = \text{Re } W p_0^* \quad (8)$$

This integrand has a pole on the real axis. This situation is caused by unrealistic assumptions, that there is no dissipation in the air. However, it is convenient to use this form and instead of introducing losses we use a small half-circle. It will be $2\pi j$ times half the residue. The same result is obtained from the Cauchy principal value (cf. [31] p 46).

The principal value is the real part of the integral. The imaginary part is unlimited. From the real part we get the power:

$$\Pi = \frac{W^2}{2\pi^2} \cdot \frac{k^2 \rho c / j}{2} \cdot \pi j \quad (9)$$

We use eq.(5) again and get the pressure at a distance z with z -polar coordinates:

$$\begin{aligned} F &= \frac{W \rho c k^3}{(2\pi)^2} \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} \frac{e^{-jk\xi z \cos \beta}}{(1 - \xi^2)} \sin \beta \xi^2 d\xi d\beta d\alpha = \\ &= \frac{W \rho c k^3}{2\pi^2} \int_0^{\infty} \frac{e^{-jkz\xi}}{(1 - \xi^2) jk\xi} \xi^2 d\xi = \frac{W \rho c k^2}{4\pi} \cdot e^{-jkz} \end{aligned} \quad (10)$$

The last step is $2\pi \cdot$ half the residue from $(1 - \xi)$, the pole in $\xi = 1$. The other factor $(1 + \xi) = 2$, $d\xi = 1$ (cf. [31] p 651 and 654).

As we see this transform is isotropical in all directions and gives the correct pressure without any complex angle path.

The three-dimensional κ -transform is however not so useful in the plane boundary case as the plane layer two-dimensional transform used previously.

It could be of interest to compare with the one- and two-dimensional cases. For the poles we use the principal values as before. We begin with two cases shown in FIG 5.10.1.



FIGURE 5.10.1. One-dimensional channel and line boundary. The line boundary is a two-dimensional point source.

Analogously we get the one-dimensional transform

$$\text{a: } \Pi = \frac{\rho c}{S} \cdot \frac{W^2 k}{2\pi} \int_{-1}^1 \frac{-j}{1 - \xi^2} \xi = \frac{\rho c}{2S} W^2 \quad (11)$$

$$\text{b: } \Pi = \rho c \frac{W^2 k}{2\pi} \int_{-1}^1 \frac{1}{\sqrt{1 - \xi^2}} d\xi = \frac{k \rho c}{2} W^2 \quad (12)$$

We also obtain the two-dimensional narrow space ($W \text{ m}^3/\text{s}$) with a two-dimensional transform.

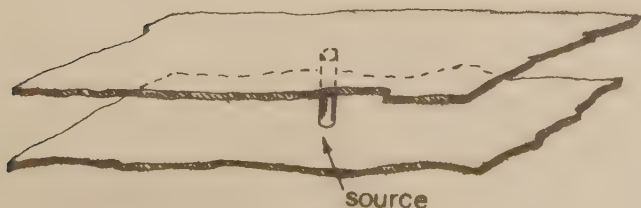


FIGURE 5.10.2. Dipole source planes.

$$\Pi = \frac{\rho c}{h} \cdot \frac{k^2}{(2\pi)} \operatorname{Re} \int_0^\infty \frac{-j/k}{1 - \xi^2} 2\pi \xi d\xi = \frac{\rho c k}{4\pi} W \quad (13)$$

We see that transforms on all coordinates lead to a pole. In such a case the transform is isotropic in direction. The three-dimensional transform is not made on a special transform axis or on the corresponding source layer plane as is the angle transform.

The latter type leads to the typical integral of ξ from 0 to 1 in all cases, e.g. on a plane in three dimensions and on a line in two dimensions. The latter case is also the same as a conphase line source transformed on a plane source layer. This case is dealt with in the following.

Jonasson [51] has treated the cylindrical waves, which are supposed to be produced by the edge of a sound barrier. He uses the same method as Ingård did for the point source.

If instead we transform on a plane parallell with the ground (v -plane), we get the following.

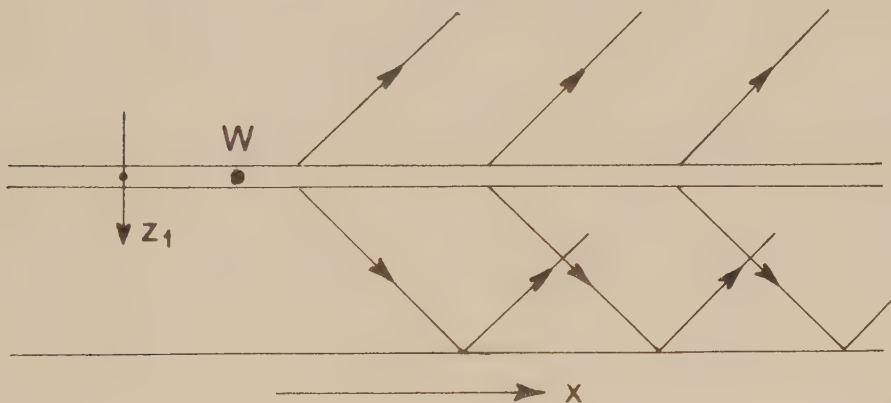


FIGURE 5.10.3. Transform source plane.

$$u_k = \int_{-\infty}^{\infty} e^{jkx} \cdot \delta(x) W dx = W \quad (14)$$

For the free line source we get:

$$p = \frac{Wk\rho c}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-jk(|z_1| \sqrt{1 - \xi^2} + kx\xi)}}{\sqrt{1 - \xi^2}} d\xi \quad (15)$$

This is obtained from the radiation of the v -layer. We formally substitute $\xi = \sin \beta$. We obtain the normalized pressure:

$$F = \int_{L=\beta_{\xi}} e^{-jk|z_1| \cos \beta} \cdot d\beta \quad (16)$$

This is Sommerfeldt's integral form of the Hankel function H_0 .

Now we can reflect the κ -wave as before and get a Q-term:

$$v = \int_{-\infty}^{\infty} \frac{2v e^{jk(\sqrt{1-\xi^2}|z+h| + \xi kx)}}{v + \sqrt{1-\xi^2}} d\xi \quad (17)$$

Evidently we get a surface wave also for a line source near the v -surface.

By using the illustrations with a radiating source plane, the limitations and the way to fulfil the boundary conditions in each case is demonstrated.

5.11. Some measurements on a surface wave on felt and a suggested method to find v of the ground

A thick felt was used to get a thin absorbent. The felt had a finite thickness resistance $R_f = \rho c r h$, which was measured in the following way:

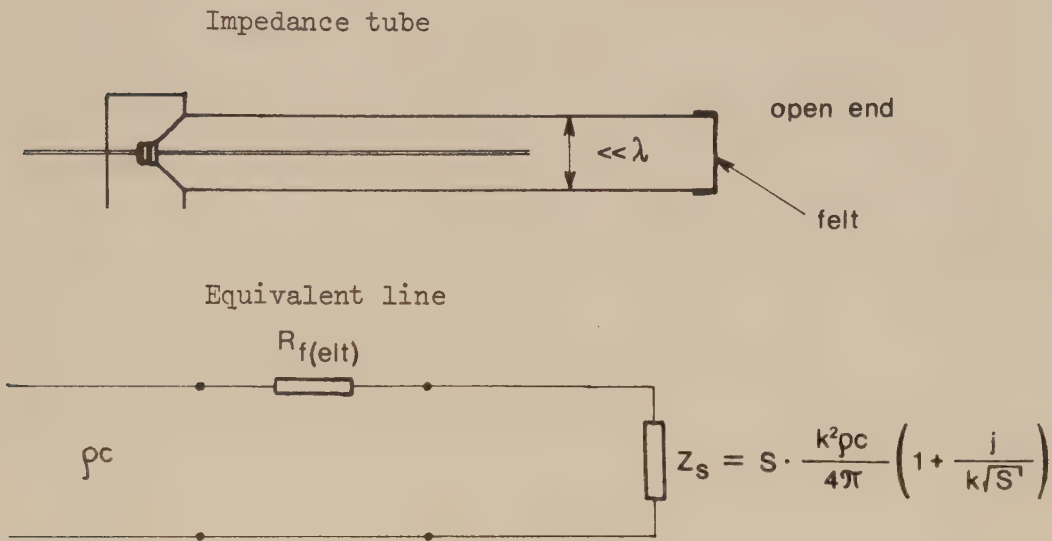


FIGURE 5.11.1. A method to find the flow resistance.

If $Sk^2 \ll 1$ we have $R_f > \text{Re}(Z_s)$ and approximately we can obtain R_f from the measured total resistance of the felt.

The value was $R_f \approx 0.63 \rho c$. (This felt has also been proposed for variable absorption purposes, as it has $R_f \approx \rho c$.) The thickness was ≈ 3.5 mm and gave a normalized flow resistance $r \approx 180$. Also the relative admittance of felt on a hard surface was measured in the conventional way in the impedance tube.

f	Rev	Imv
1000	0.035	0.022
2670	0.16	0.035

Table 5.11.1. The relative admittance of felt.

At 2 kHz v is large enough to affect also the space wave. Some frequency scannings are shown below.

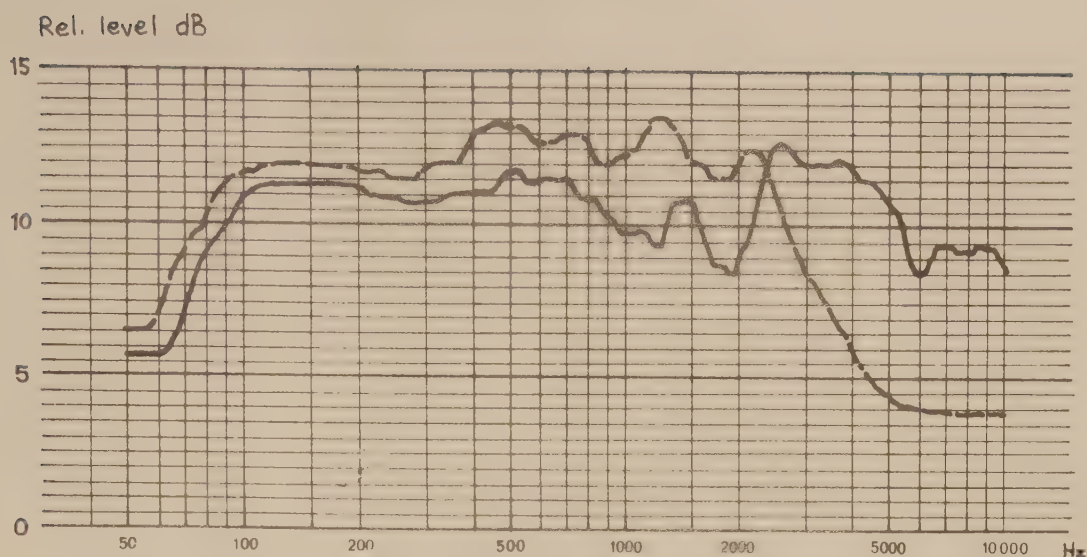


FIGURE 5.11.2. Frequency scannings. — 1 m
--- 0 m

At the frequency with the largest surface value we made a height scanning.



FIGURE 5.11.3.

According to eq.(5.4.6) the attenuation shall be $8.7 \cdot \text{kmv dB/m}$. This gives $v \approx 30/(8.7 \cdot f/55) = 0.15$.

As it is very difficult to measure in v , this pressure gradient method is suggested for impedance measurements on e.g. a grass ground. The difficulties arise from the fact that the exact location of the first minimum in a standing wave must be given as a distance to the surface. The surface location cannot possibly be thoroughly known, because of the grass, the roughness etc.

On the other hand, it is possible to determine the standing wave ratio S .

by an impedance tube pressed down in the earth or by a free standing wave. This gives $|B|$, B = reflexion factor.

$$|B| = \frac{S - 1}{S + 1} \quad (1)$$

$$B = \frac{1 - v}{1 + v} = 1 - \frac{2}{1 + \frac{2}{v}} \quad (2)$$

$$|B| = \frac{(1 - \operatorname{Re}(v))^2 + (\operatorname{Im}(v))^2}{(1 + \operatorname{Re}(v))^2 + (\operatorname{Im}(v))^2} \quad (3)$$

This can be evaluated directly from a Smith chart:

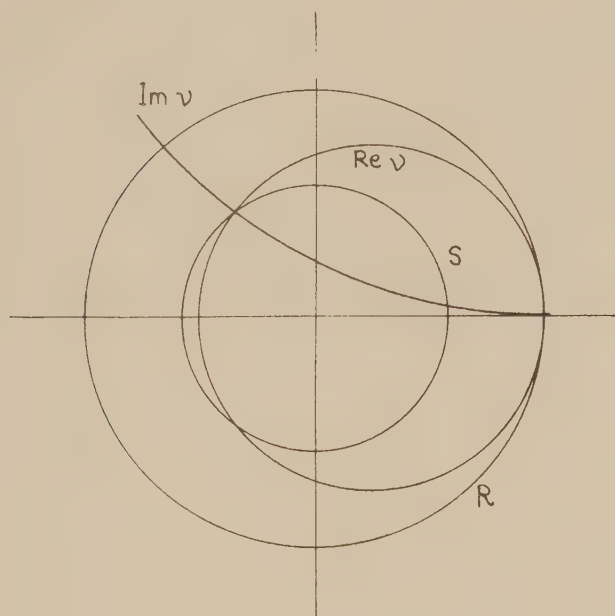


FIGURE 5.11.4. Impedance diagram.

We have been working on the problem of finding v for a ground but failed with all types of standing waves and also with a direct impedance measurement from pressure and velocity at the surface. I hope that the sketched method will solve our problems.

A remarkable thing is the account in [60], that the ground impedance has been measured very accurately. This might have been due to a perfect English lawn, where the surface location was known. However, there is

also used impedances according to the $(-j\omega t)$ -convention and impedance diagrams according to $+j\omega t$, which I find very puzzling. In a letter to Professor P.E. Doak three years ago I brought up this problem, but I never got an answer to my question.

5.12. Concluding remarks to Part 5

The point source on a surface of absorbing type gives a surface wave, which has been shown and studied in its simplest form.

The transforms used by Sommerfeldt and Weyl have been shown to give different results and formally the second method does not fulfil the boundary conditions.

By using transform source layers a simple picture of transform and boundary mutual connections has been introduced. This also gives the real original integrals used by Sommerfeldt.

PART 6

MEAN SQUARE MEASUREMENTS AND SPATIAL CORRELATION

6.1. Introduction

The classical diffuse field model is composed of travelling plane waves. This leads to simple space correlation functions for the sound pressure.

The space correlation of the mean square is often more interesting, as microphone signals are squared and integrated before space averaging. It seems as if some covariance functions often used for the mean square, fail to be correct or are improperly used. I therefore introduce a new model to get the covariance of the mean square in a mode and for almost diffuse fields. This is necessary as a travelling wave gives the covariance ≈ 0 for the mean square in the space.

The new covariance function for the mean square is of a similar type as for the pressure in plane waves, but more complicated and with different wavenumbers depending on the mode type for different directions. The covariance is therefore approximately evaluated for some special cases and compared with measurements. The result is used in a brief discussion about the advantages of discrete versus continuous spatial averaging.

6.2. The variance of the mean square measurement

In measurements the aim is usually to minimize the variance of the measured value. The value is usually obtained from a discretely or continuously averaging system. In acoustics both time and space averaging are frequently used.

We start with the time averaging. The variance of the measured average value can be expressed in the variance σ and the covariance C . C describes the statistical dependence, caused by the physical system:

$$C_{mn} = E[(x_m - \mu)(x_n - \mu)] \quad (1)$$

C is related to the correlation:

$$C_{mn} = R_{mn} - \mu^2 \quad (2)$$

The variance of the measured value will be used in the following, and we shall briefly show the connections:

$$y = \frac{1}{N} \sum x_n$$

$$\mu = E[x] = E[y] \quad \} \quad (3)$$

$$\sigma_x^2 = E[(x - \mu)^2]$$

$$\sigma_y^2 = E[(y - \mu)^2] = E\left[\left(\frac{1}{N} \sum_1^N x_n - \mu\right)^2\right] =$$

$$= \frac{\sigma^2}{N} + E\left[\frac{2}{N^2} \sum_{m=1}^N \sum_{n=m+1}^N (x_m - \mu)(x_n - \mu)\right] =$$

$$= \frac{\sigma^2}{N} + \frac{2}{N^2} \sum_{m=1}^N \sum_{n=m+1}^N C_{mn} \quad (4)$$

C_{mn} is given by the properties of the physical system. This information can be used to minimize σ_y .

For mean square measurements with discrete sampling we insert $q^2 = x$ in eq.(4). More common is continuous averaging giving the following integral:

$$y = \bar{x} = \frac{1}{T} \int_0^T x(t) dt \quad (5)$$

If the time averaging is continuously performed $(t - T, t)$, $\overline{q^2} = D$ is a function of time:

$$D(t) = \frac{1}{T} \int_{t-T}^t q^2(t) dt \quad (6)$$

The auto-correlation of $D(t)$ is

$$R_D(\tau) = E[D(t) \cdot D(t - \tau)] \quad (7)$$

The product of two integrals is rewritten as a double integral:

$$\begin{aligned} R_D(\tau) &= E \left[\frac{1}{T^2} \int_0^T \int_0^T q^2(\xi) q^2(\eta - \tau) d\eta d\xi \right] = : \\ &= \frac{1}{T^2} \int_0^T \int_0^T E \left[q^2(\xi) q^2(\eta - \tau) \right] d\eta d\xi = \\ &= \frac{1}{T^2} \int_0^T \int_0^T R_{q^2}(\xi - \eta + \tau) d\eta d\xi = \\ &= \frac{1}{T} \int_{-T}^T \left(1 - \frac{|\zeta|}{T} \right) R_{q^2}(\zeta + \tau) d\zeta \end{aligned} \quad (8)$$

As $\mu_D = \mu_{q^2}$ we also get

$$C_D(\tau) = \frac{1}{T} \int_{-T}^T \left(1 - \frac{|\zeta|}{T} \right) C_{q^2}(\zeta + \tau) d\zeta \quad (9)$$

and

$$\sigma_D^2 = C_D(0) = \frac{1}{T} \int_{-T}^T \left(1 - \frac{|\tau|}{T}\right) C_{q^2}(\tau) d\tau \quad (10)$$

The simplification from double to single integral is possible, as the two variables only form a difference in the function R_{q^2} .

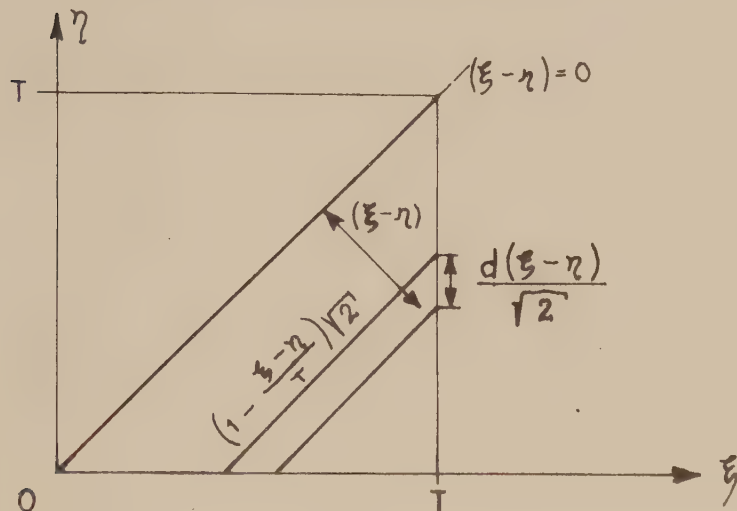


FIGURE 6.2.1. Integration area.

We revert to the discrete sampling, but with constant sample increment $\Delta\tau$. We rewrite eq.(4) with $y = q^2 = D$ and $x = q^2$. With $C_{mn} = C(t_m - t_n) = C(\tau_v) = C(v\Delta\tau)$ we obtain:

$$\sigma_D^2 = \frac{\sigma_{q^2}}{N} + \frac{2}{N} \sum_1^N \left(1 - \frac{v}{N}\right) C_{q^2}(v\Delta\tau) \quad (11)$$

Now we shall study a relation only valid for Gaussian noise. If $q_1(t)$ and $q_2(t)$ are normally distributed and have the expected value $\mu_q = 0$, we get according to [53] p.92:

$$\begin{aligned} R_{q^2,1,2} &= E[q_1^2 \cdot q_2^2] = E[q_1^2] \cdot E[q_2^2] + 2 \left(E[q_1 \cdot q_2]\right)^2 = \\ &= \sigma_q^4 + 2R_{q,1,2}^2 = \mu_{q^2}^2 + 2R_{q,1,2}^2 \end{aligned} \quad (12)$$

With $q_1 = q(t_1)$ and $q_2 = q(t_1 + \tau)$ we get:

$$R_{q^2}(\tau) = \mu_{q^2}^2 + 2R_q^2(\tau) \quad (13)$$

We also have the usual connection:

$$R_{q^2} - \mu_{q^2}^2 = C_{q^2} \quad (14)$$

Therefore we also obtain:

$$C_{q^2} = 2R_q^2 = 2C_q^2 \quad (15)$$

This is a remarkable result as R_q^2 is always positive. Consequently C_q^2 is always positive as well. For Gaussian noise the covariance of q^2 is always positive. This can be explained by the very special probability density function of q^2 . For variables that are symmetrical about μ , C normally changes sign.

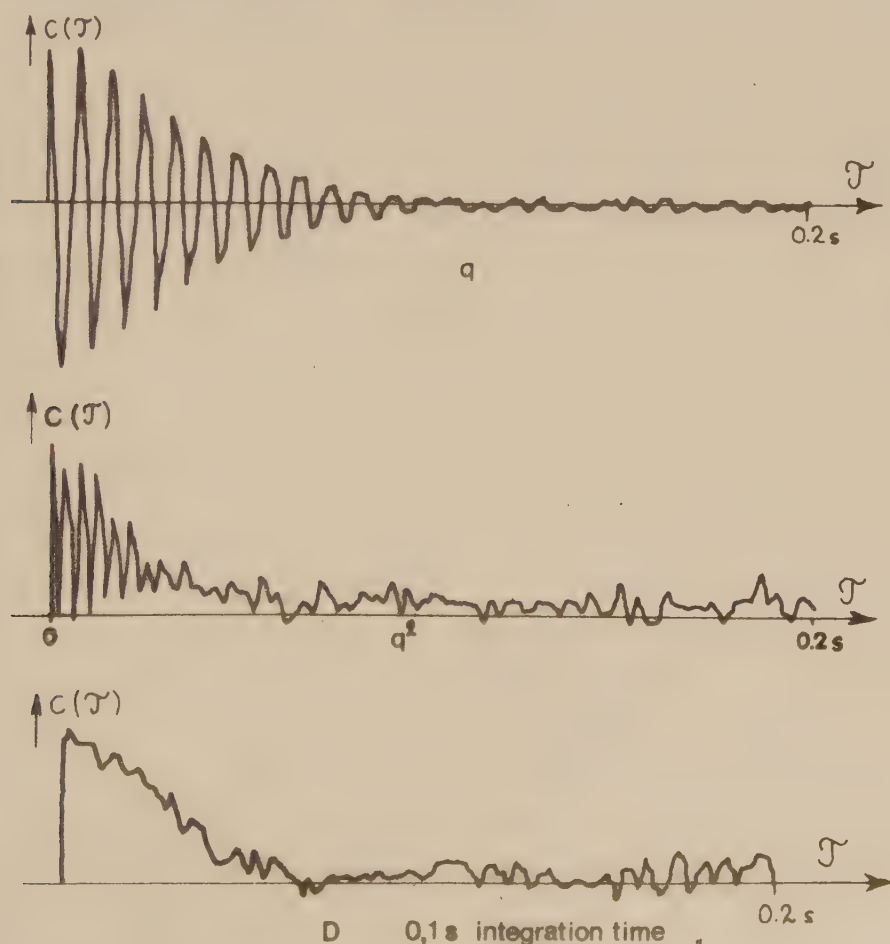


FIGURE 6.2.2. Covariance in the time domain of q , q^2 and $\overline{q^2} = D$, 10 Hz noise band at 125 Hz.

We insert eq.(15) in eq.(9) and get

$$\sigma_D^2 = \frac{4}{T} \int_0^T \left(1 - \frac{\tau}{T}\right) R_q^2(\tau) d\tau \quad (16)$$

The "amplitude" of R is usually decreasing, so we can use the following condition:

$$R(\tau) \ll R(0) = \sigma_q^2 = \mu_q^2 \quad \text{for } \tau > \tau_1 \quad (17)$$

If $T \gg \tau_1$ the integral is still dominated by the part with $\tau < \tau_1$ and we can omit τ/T . Furthermore, the integral can be extended to ∞ :

$$\sigma_D^2 \approx \frac{1}{T} \int_{-\infty}^{\infty} C_q^2(\tau) d\tau = \frac{2}{T} \int_{-\infty}^{\infty} R_q^2(\tau) d\tau = \frac{2}{T} \int_{-\infty}^{\infty} C_q^2(\tau) d\tau \quad (18)$$

In [53] the condition for (8) is that $T > |\tau|$, but τ is the integration variable. It ought to be a relation to some quality of $R(\tau)$, that gives the condition as stated above.

6.3. The correlation and variance for filtered white noise before and after the room

The correlation of filtered white noise is directly obtained from $|H(\omega)|^2$ of the filter. We assume an ideal filter with the bandwidth B_F and the center frequency f_0 . We put $2\omega_1 = 2\pi B$, $\omega_0 = 2\pi f_0$ and $s(\omega) = \mu_q 2/B_F$:

$$R_q = \frac{\mu_q 2}{2\pi B_F} \operatorname{Re} \int_{\omega_0 - \omega_1}^{\omega_0 + \omega_1} e^{j\omega\tau} d\omega = \mu_q 2 \cos(\omega_0 \tau) \frac{\sin(\omega_1 \tau)}{\omega_1 \tau} \quad (1)$$

If $T \gg \tau_1 = 5/B$, eq.(6.2.17) can be used and we get:

$$\begin{aligned} \epsilon_D^2 &= \frac{\sigma_D^2}{\mu_D^2} = \frac{4}{T} \int_0^\infty \cos^2(\omega_0 \tau) \frac{\sin^2(\omega_1 \tau)}{(\omega_1 \tau)^2} d\tau \approx \\ &\approx \frac{2}{T} \int_0^\infty \left(\frac{\sin(\omega_1 \tau)}{\omega_1 \tau} \right)^2 d\tau \approx \frac{2}{T} \cdot \frac{1}{2} \cdot \frac{\pi}{\omega_1} = \frac{1}{B_F T} \end{aligned} \quad (2)$$

This is a common expression for estimation of ϵ_D or the needed T to get a certain ϵ_D .

For a single resonant system like a mode with the half-power bandwidth $B_{0.5}$ the following covariance and ϵ_D is obtained. We put $f = 2\pi(\omega_0 + \omega)$ and $B/2 = 2\omega_2$. The filter peak is $1/(1 + (\omega/\omega_2)^2)$. We normalize to get γ_q and split the denominator, to be able to use the upper pole at $\omega = j\omega$. The residue gives the solution. We have to use different closed loops for $\tau > 0$ and $\tau < 0$ but put $|\tau|$ in the solution.

$$\gamma_q = \frac{2/\omega_2}{2\pi} \operatorname{Re} \int_{\text{uhp}} \frac{e^{j(\omega_0 + \omega)\tau}}{1 + (\omega/\omega_2)^2} d\omega = \cos(\omega_0 \tau) \cdot e^{-\omega_2 |\tau|} \quad (3)$$

Hence the error in D is:

$$\epsilon_D^2 = \frac{2}{T} \int_{-\infty}^{\infty} \cos^2(\omega_0 \tau) \cdot e^{-\omega_2 |\tau|} \cdot d\tau \approx \frac{2}{T\omega_2} = \frac{2}{T\pi B_{0.5}} \quad (4)$$

It should be noticed that $1/B_F T$, where B_F is the filter bandwidth, only gives ϵ_D if the transfer function of the whole system is mainly determined by the filter. In a room with few modes the total $H(\omega)$ can be determined by the room within the filter band. With one mode only, $\epsilon_D = 1/(T\pi B_{0.5}/2)$, which is an extreme case and gives much longer measuring time than expected from $1/(B_F T)$. With many modes, $|H(\omega)|$ is more smooth and determined mainly by the filter.

Below are shown recordings of $R_q(\tau)$ before and after a small room at low frequency.

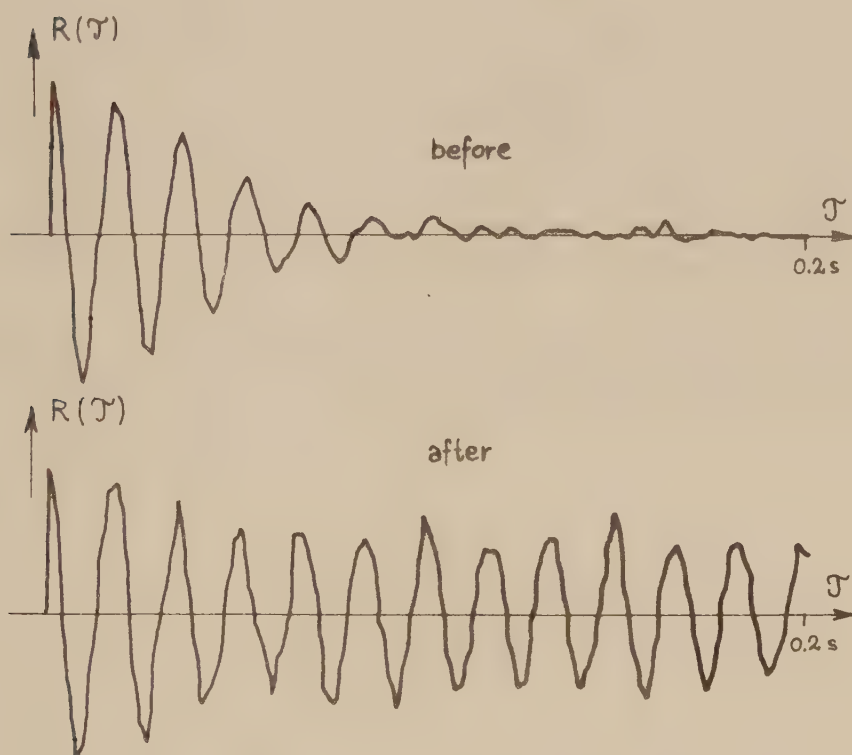


FIGURE 6.3.1. Great influence on $R_q(\tau)$. 30 m^3 , 60 Hz - 70 Hz.

We also made recordings in the reverberant room at 500 Hz. These recordings gave a small difference between R_q -curves before and after the room.



FIGURE 6.3.2. Small difference of $R_q(\tau)$ before and after the room
 (200 m^3 , $f_0 = 500 \text{ Hz}$, $B_F = 10 \text{ Hz}$).

It is evident that $R_q(\tau)$ decreases faster with many modes than with few. For a single mode $R_q(\tau)$ decreases as $\exp[-\tau\pi F_{0.5}]$, but for many modes the decrease is determined by B_F . The reason why $R_q(\tau)$ can decrease faster than the reverberation is the different phase lags of different modes (cf. [55]).

Hence the room is a much more efficient channel of information, as it is ready for new information after the decrease time τ_1 of $R(\tau)$. ($R(\tau) < \text{small value for } \tau > \tau_1$).

6.4. Spatial correlation in travelling wave fields

Now we shall turn to the correlation $R(\tau, \vec{r})$ with $\tau = 0$ and $\vec{r} = \vec{r}_2 - \vec{r}_1$. $\vec{r}_1$ and $\vec{r}_2$ are coordinates in the space.

The correlation at a certain distance and direction r is:

$$R_q(\vec{r}) = E[q(\vec{r}_1), q(\vec{r}_2)] \quad (1)$$

For a plane, pure tone wave we get the following γ_q , the correlation normalized on the variance σ_q^2 . The angle between $\vec{r}$ and the wave propagation is ϕ :

$$\gamma_q(\vec{r}) = \frac{R_q}{\sigma_q^2} = 2E[\cos(\omega t) \cdot \cos(\omega t - kr \cos \phi)] = \cos(kr \cos \phi) \quad (2)$$

As $\mu_q = 0$, $R_q = C_q$. The normalized covariance is $C_q/\sigma_q^2 = \delta_q$. With band limited white noise we get the following, if the wave source is a plane vibrating surface excited by the noise signal. In a plane wave we have a perfect impulse response $h(t) = \delta(t - r \cos \phi/c)$. Therefore we can use eq.(6.3.1) also in the space, for a plane wave produced by filtered noise. Then $\tau = r \cos \phi/c$ and we only change $\omega_0 \tau$ for $k_0 r \cos \phi$ and $\omega_1 \tau$ for $k_1 r \cos \phi$.

$$R_q = \mu_q^2 \cdot \cos(k_0 r \cos \phi) \frac{\sin(k_1 r \cos \phi)}{k_1 r \cos \phi} \quad (3)$$

If we let $\vec{r}$ have all directions in a plane containing the wave direction, we get for the pure tone wave:

$$\gamma_q(r) = \frac{1}{2\pi} \int_0^{2\pi} \cos(kr \cos \phi) d\phi = J_0(kr) \quad (4)$$

If we let $\vec{r}$ have all three-dimensional directions instead we get:

$$\gamma_q(r) = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi \cos(kr \cos \phi) \sin \phi d\phi d\psi = \frac{\sin kr}{kr} \quad (5)$$

The last expression can also be the result in one direction or in all directions, if we have diffuse fields according to the classical travelling wave idea. Then the field is the sum of waves of random phases α_n , directions ψ_n and amplitudes p_n . In two dimensions we get γ according to the following:

$$\begin{aligned}\gamma_q(\vec{r}) &= \frac{1}{\mu_{q2}} E \left[(\bar{z} p_m \cos(\omega t + \alpha_m)) (\Sigma p_n \cos(\omega t + \alpha_n - kr \cos \psi_n)) \right] = \\ &= \frac{1}{\mu_{q2}} E \Sigma \left[p_m^2 (\cos(\omega t + \alpha_m) \cos(\omega t + \alpha_m - kr \cos \psi_n)) \right] = \\ &= J_0(kr)\end{aligned}\tag{6}$$

α_n has been supposed to give cancelling out in the average of all cross products for $m \neq n$.

In one direction or all directions in a three-dimensional field we get in the same manner:

$$\gamma_q(r) = \frac{\sin kr}{kr}\tag{7}$$

These expressions are very well known and are given in [56] by Cook et.al.

The pressure squared (q^2) gives a new "wave" with $\mu_{q2} \neq 0$. The trigonometrical relation is:

$$\cos^2(\omega t - kx) = \frac{1}{2} (1 + \cos(2\omega t - 2kx))\tag{8}$$

q^2 is a "wave" with the double frequency and double wavenumber. The mean value of q^2 is 0.5 times the original (amplitude)² and equal to the " q^2 -amplitude". We also realized that $\sigma_{q2}^2 = \mu_{q2}^2/2$, as the deduction of σ^2 means another squaring procedure. As $\mu_{q2} \neq 0$ we have to distinguish between R and C. For the sinusoidal distribution we evidently get the following in both time and space (pure tone and pure tone wave):

$$\text{Tone: } \sigma_{q^2} = \mu_{q^2}^2/2 = \sigma_q^2/2 \quad (9)$$

We recollect that for Gaussian noise we had:

$$\text{Noise: } \sigma_{q^2}^2 = 2\sigma_q^4 \quad (10)$$

and

$$\text{Noise: } C_{q^2} = 2R_q^2 \quad (11)$$

We directly get the following expression in one direction for a plane pure tone wave. The covariance normalized on the variance is:

$$\delta_{q^2} = \cos(2kr \cos \psi) \quad (12)$$

In this case the covariance of q^2 has sign shifts.

Furthermore, we get for all directions in the wave propagation plane of a pure tone wave:

$$\text{Tone: } \delta_{q^2} = J_0(2kr) \quad (13)$$

For all directions of a pure tone wave we obtain:

$$\delta_{q^2} = \frac{\sin 2kr}{2kr} \quad (14)$$

For normally distributed noise eqs.(3) and (11) give:

$$\text{Noise: } \delta_{q^2} = \delta_q^2 = \cos^2(k_0 r \cos \phi) \frac{\sin^2(k_1 r \cos \phi)}{(k_1 r \cos \phi)^2} \quad (15)$$

$$\text{Noise: } C_{q^2}(r) = 2R_q^2(r) \quad (16)$$

For all directions in one Gaussian plane wave we get:

$$\begin{aligned}
 R_q &= E[q_1(t) \cdot q_2(t)] = E\left[q_1(t) q\left(t - \frac{r \cos \phi}{c}\right)\right] = \\
 &= \sigma_q^2 E\left[\cos\left(k_o r \cos \phi \frac{\sin(k_1 r \cos \phi)}{k_1 r \cos \phi}\right)\right] \approx \\
 &\approx \sigma_q^2 \frac{\sin(k_o r)}{k_o r} ; \quad \delta_q = \gamma_q \approx \frac{\sin(k_o r)}{k_o r} \quad (17)
 \end{aligned}$$

The last step is valid for $k_1 r \ll 1$ ($\sin k_1 r \approx k_1 r$). This means small distances compared to the "bandwidth wavelength".

As a linear system does not alter the distribution of Gaussian noise we obtain the covariance of q^2 from the covariance of q :

$$\delta_{q^2} \approx \left(\frac{\sin(k_o r)}{k_o r} \right)^2 \quad (18)$$

We get an analogous solution for three dimensions, with a field built up by a statistically independent plane noise wave s , as all cross products cancel out:

$$\begin{aligned}
 E[(\Sigma q_m)(\Sigma q_n)] &\approx \sigma_q^2 \frac{\sin(k_o r)}{k_o r} + \mu_{q^2} \\
 E[(\Sigma q_m^2)(\Sigma q_n^2)] &\approx \mu_{q^2}^2 \left(1 + 2 \left(\frac{\sin(k_o r)}{k_o r} \right)^2 \right) \quad \} \quad (19) \\
 \text{Noise: } \delta_{q^2} &\approx \left(\frac{\sin k_o r}{k_o r} \right)^2
 \end{aligned}$$

The simple relations depend on the "perfect" transfer function between two points in a travelling wave. The signal is only arriving later,

further down in the field, $(h(t) = \delta(t - r \cos \phi / c)$ and $H(\omega) \approx \exp(-jk \ r \cos \phi)$.

The covariance $C_{q^2}(\tau)$ was used to give the standard error of $D(T)$. The covariance $C_q(r)$ and $C_{q^2}(r)$ consequently can be used to estimate the variance of the mean values of q and q^2 measured in space. The first case is valid if many microphones are directly connected, and the latter if the signals are added after individual squaring devices, but before integration.

However, the common method is to time average q^2 to get the mean square in each point $\overline{q^2}$. After that a space mean value is formed. The correlation of $\overline{q^2} = D$ in one direction of a plane wave Gaussian field is:

$$\begin{aligned} R_D &= E\left[\overline{q_1^2} \cdot \overline{q_2^2}\right] = E\left[\frac{1}{T^2} \int_0^T \int_0^T q^2(\xi) q^2\left(\xi - \frac{r}{c} \cos \phi\right) d\xi d\tau\right] = \\ &= \frac{2}{T} \int_0^T \left(1 - \frac{\tau}{T}\right) R_{q^2}\left(\tau + \frac{r}{c} \cos \phi\right) d\tau \end{aligned} \quad (20)$$

$$\begin{aligned} \text{Noise: } C_D(r) &\frac{2}{T} \int_0^T \left(1 - \frac{\tau}{T}\right) C_{q^2}\left(\tau + \frac{r}{c} \cos \phi\right) d\tau = \\ &= \frac{4}{T} \int_0^T \left(1 - \frac{\tau}{T}\right) R_q^2\left(\tau + \frac{r}{c} \cos \phi\right) \cdot d\tau \end{aligned} \quad (21)$$

$R_q^2(\tau) \rightarrow 0$ for $\tau \rightarrow \infty$, and $C_D(r) \rightarrow 0$ for long measuring time. $D \rightarrow \mu_{q^2} = \theta$ and the covariance of the point expected value θ is $C_\theta(r) \equiv 0$.

This is a quite natural result, as the energy density is the same in all points of a field of travelling waves, if the waves are uncorrelated.

We must therefore use a standing wave or mode model to get an idea about $C_\theta(r)$ in real room fields. $C_\theta(r)$ can be used to estimate the standard error of a space mean of θ in the same manner as $C_{q^2}(\tau)$ for D in a point and $C_{q^2}(r)$ for q^2 in a wave field.

6.5. The correlation of θ in 1-, 2- and 3-modes

θ is the expected point value of q^2 . The space expected value is denoted $\mu_\theta = \mu_{q^2}$. We shall confine our study to the spatial correlation of θ .

$$C_\theta(r) \approx E[(\theta(\vec{r}_1) - \mu_\theta)(\theta(\vec{r}_1 + \vec{r}\beta) - \mu_\theta)] = R_\theta(r) - \mu_\theta^2 \quad (1)$$

(β is the direction of $\vec{r}$.)

A room field can be regarded as a sum of modes. We refer to the modes as 1-, 2- and 3-modes, as they apply to 1, 2 and 3 dimensions. A mode can be regarded as a sum of correlated travelling waves. For one direction of $\vec{r}$ in a 1-mode we obtain:

$$\begin{aligned} R_\theta(r) &= 4\mu_\theta^2 = E[\cos^2(kx) \cdot \cos^2(kx + r \cos \phi)] = \\ &= \mu_\theta^2 \left(1 + \frac{1}{2} \cos(2kr \cos \phi)\right) \end{aligned} \quad (2)$$

The normalized covariance in one direction is:

$$\delta_\theta = \cos(2kr \cos \phi) \quad (3)$$

For all directions we get:

$$\delta_\theta = \frac{\sin(2kr)}{2kr} \quad (4)$$

For a pure tone a mode has the same phase ($\pm$) in all points, so the momentary observations give the same result as integrals:

$$\text{Tone: } C_{q^2}(r) = C_\theta(r) \quad (5)$$

$$\text{Noise: } C_{q^2}(r) \neq C_\theta(r)$$

The pressure correlation is the same as for travelling waves. For all directions of $\vec{r}\beta$ in a 1-mode we obtain:

$$\delta_q = \frac{\sin(kr)}{kr} \quad (6)$$

Θ of a 2-mode can be given in the following form:

$$\begin{aligned} \Theta &= \mu_\Theta (1 + \cos(2k_x x))(1 + \cos(2k_y y)) = \\ &= \mu_\Theta (1 + \cos(2k_x x) + \cos(2k_y y) + \cos(2k_x x) \cdot \cos(2k_y y)) \end{aligned} \quad (7)$$

As the product is separable in x -, y - and z -dependence we start by deducing R_Θ . C_Θ is later obtained from:

$$C_\Theta = R_\Theta - \mu_\Theta^2 \quad (8)$$

Using the product sign Π and the coordinates $\vec{r} = [\xi_v]$ we get the following expression for a special direction $\Delta\vec{r}$:

$$\Delta\vec{r} = [\Delta\xi_v] \quad (9)$$

$$\frac{R_\Theta}{\mu_\Theta^2} = E \left[\frac{M}{\Pi} (1 + \cos(2k_v \xi_v))(1 + 2k_v(\xi_v + \Delta\xi_v)) \right] \quad (10)$$

The coordinates are orthogonal and the factors are independent, so we can take $E[\]$ one by one:

$$\begin{aligned} E \left[(1 + \cos(2k_v \Delta\xi_v))(1 + \cos(2k_v(\xi_v + \Delta\xi_v))) \right] &= \\ &= 1 + \frac{1}{2} \cos(2k_v \Delta\xi_v) \end{aligned} \quad (11)$$

Thus the total correlation will be the following:

$$\frac{R_\Theta(\Delta\vec{r})}{\mu_\Theta^2} = \frac{M}{\Pi} \left(1 + \frac{1}{2} \cos(2k_v \Delta\xi_v) \right) \quad (12)$$

The variance is:

$$\sigma_{\theta}^2 = C_{\theta}(0) = R_{\theta}(0) - \mu_{\theta}^2 = \mu_{\theta}^2 \left(\left(1 + \frac{1}{2}\right)^M - 1 \right) \quad (13)$$

The normalized standard error for $M = 1, 2, 3$ is:

$$\epsilon^2(0) = \frac{\sigma_{\theta}^2}{\mu_{\theta}^2} = \frac{1}{2} = 0.5 \quad \text{for a 1-mode} \quad (14)$$

$$\epsilon^2(0) = 1.5^2 - 1 = 1.25 \quad \text{for a 2-mode} \quad (15)$$

$$\epsilon^2(0) = 1.5^3 - 1 = 2.375 \quad \text{for a 3-mode} \quad (16)$$

$$\mu_{\theta} = \frac{\theta_{\max}}{2^M} \quad (17)$$

The range of θ is $(0, \theta_{\max})$ and the distribution is unsymmetrical for $M \geq 2$. It is only for 1-modes that peaks and valleys are equal. FIG 6.5.1 shows point peaks and line valleys for θ in a 2-mode. The diminishing μ_{θ} for the same interval of θ explains the increasing ϵ with increasing M .

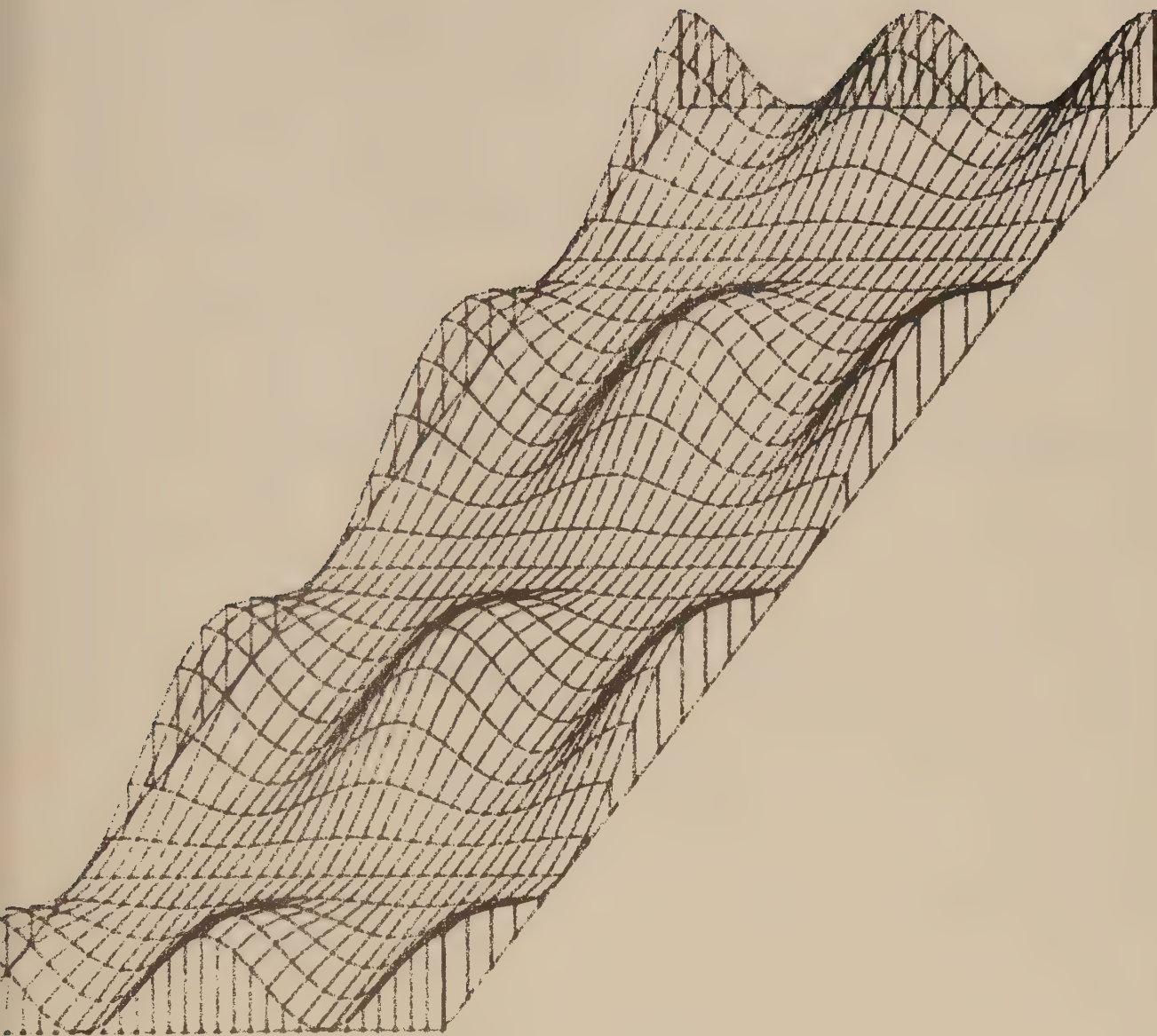


FIGURE 6.5.1. Field picture.

6.6. The covariance of all directions of r

We revert to eq.(6.5.12) and perform the product for $M = 3$:

$$\begin{aligned} \frac{R_{\theta}}{2} = 1 + \frac{1}{2} \sum_1^3 \cos(2k_{\nu} \Delta \xi_{\nu}) + \frac{1}{4} \sum_{\mu \neq \nu} \cos(2k_{\mu} \Delta \xi_{\mu}) \cos(2k_{\nu} \Delta \xi_{\nu}) \\ + \frac{1}{8} \prod_1^3 \cos(2k_{\nu} \Delta \xi_{\nu}) \end{aligned} \quad (1)$$

For $M = 2$ we get:

$$\frac{R_{\theta}}{2} = 1 + \frac{1}{2} (\cos(2k_1 \Delta \xi_1) + \cos(2k_2 \Delta \xi_2)) + \frac{1}{4} (\cos(2k_1 \Delta \xi_1) \cos(2k_2 \Delta \xi_2)) \quad (2)$$

We introduce $\vec{\beta}$, the direction of $\Delta \vec{r}$:

$$\begin{aligned} \delta(\Delta \vec{r}) = \frac{2}{5} (\cos(2k \Delta r \vec{\beta} \vec{\xi}_1) + \cos(2k \Delta r \vec{\beta} \vec{\xi}_2)) + \\ + \frac{1}{5} \cos(2k \Delta r \vec{\beta} \vec{\xi}_1) \cos(2k \Delta r \vec{\beta} \vec{\xi}_2) \end{aligned} \quad (3)$$

The first terms give directly the same integrals as in a 1-mode, so for all directions we obtain:

$$\delta_{\theta}(r) = \frac{2}{5} \left(\frac{\sin(2k_1 r)}{2k_1 r} + \frac{\sin(2k_2 r)}{2k_2 r} \right) + \frac{1}{5} (\delta A(k_1, k_2, \phi) d\phi) \quad (4)$$

The last term is small, so we can get an approximate δ_p^2 from the first two terms. A is a symbol for the rest terms.

In the same manner we get for $M = 3$:

$$\begin{aligned} \delta_{\theta} = \frac{4}{19} \sum \cos(2k_{\nu} \Delta \xi_{\nu}) + \frac{2}{19} \sum_{\mu \neq \nu} (\cos(2k_{\nu} \Delta \xi_{\mu})) \cos(2k_{\nu} \Delta \xi_{\nu}) + \\ + \frac{1}{19} \prod_1^3 \cos(2k_{\nu} \Delta \xi_{\nu}) \end{aligned} \quad (5)$$

After integration over all directions we obtain:

$$\delta_0 = \frac{4}{19} \left(\sum_1^3 \frac{\sin(2k_v r)}{2k_v r} \right) + \frac{6}{19} \iint A_1 \sin \psi \, d\psi d\phi + \frac{1}{19} \iint A_2 \sin \psi \, d\psi d\phi \quad (6)$$

It is now possible to form an average over the k_v -combinations for a given frequency range, at least by a computer. To get a simple approximate analytical expression we shall instead use what is considered typical modes.

If we put all k_v equal we get a "diagonal" mode, which we assume to be typical for the mode type. If all k_v are equal we get:

$$M = 2: \quad k_v = k/\sqrt{2} \quad (7)$$

$$M = 3: \quad k_v = k/\sqrt{3} \quad (8)$$

The factors of A , A_1 and A_2 vary in the following typical manner obtained for A :

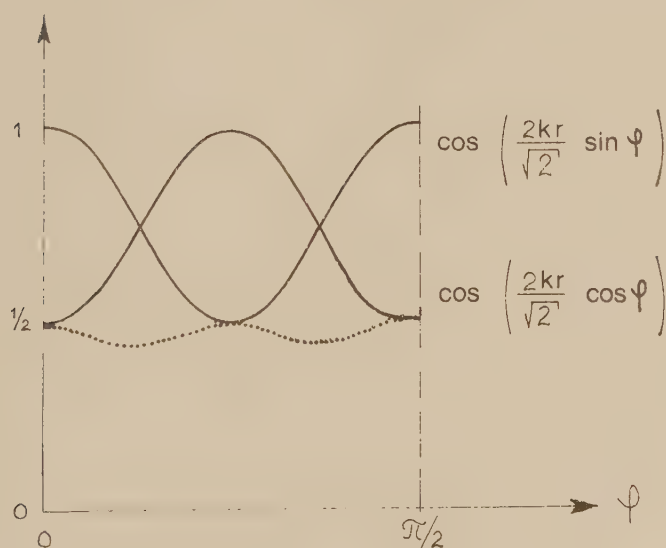


FIGURE 6.6.1. The factors in A .

The product is always less than one of the factors, but as a rough estimation we only take the factor and get:

$$M = 3: \quad \delta_{\theta}(r) \approx \frac{\sin(2kr/\sqrt{3})}{2kr/\sqrt{3}} \quad (9)$$

$$M = 2: \quad \delta_{\theta}(r) \approx \frac{\sin(2kr/\sqrt{2})}{2kr/\sqrt{2}} \quad (10)$$

6.7. Covariance of the mean square in the complete field

Let us start with a simple example, two modes in the same direction with slightly different k -numbers, k_1 and k_2 :

$$\frac{R_{\theta}}{\mu_{\theta}^2} = 1 + \frac{1}{4} \left(\cos\left((k_1 + k_2) \frac{x}{2}\right) \right) \left(\cos\left((k_1 - k_2) \frac{x}{2}\right) \right) \quad (1)$$

If $k_1 \approx k_2$ we get for $(k_1 - k_2)x \ll 1$:

$$\frac{R_{\theta}}{\mu_{\theta}^2} = 1 + \frac{1}{4} \cos(2k\Delta x) \quad (2)$$

$$\epsilon_{\theta}^2 = \frac{1}{4} \quad (3)$$

$$\delta(x) = \cos(2k\Delta x) \quad (4)$$

One mode gave a factor $\epsilon_{\theta}^2 = \frac{1}{2}$. We get a smaller standard error for the increasing number of modes, N , but the covariance normalized on the variance is the same as for one mode, if the relative frequency bandwidth is small.

We assume that N modes in the band with relative mode distances $\gg 1$ are driven by different noise intervals of the ω -axis. Thus there are N statistically independent modes:

$$\frac{\sigma_{\theta}^2}{\mu_{\theta}^2} = \epsilon_N = \frac{\epsilon_{\theta 1}}{N_1} < \frac{\epsilon_{\theta 1}}{(B_{\text{filter}}/B_{\text{mode}})} \quad (5)$$

$$\delta_{\theta N} = \text{average of } \delta_{\theta 1} \quad (6)$$

The limitation in eq.(5) is given by the fact that with modal overlap there are only N_2 free intervals: $N_2 = B_{\text{filter}}/B_{\text{mode}}$.

The loudspeaker is coupling in the same way to the mode by the factor $P_n(\vec{r}_0)$ as the observation is coupled to the mode by $P_n(r)$. For full modes we get a total relative error $(1.5^6 - 1)$ instead of $(1.5^3 - 1)$ when the variation between modes is included ($N = N_1$ or N_2):

$$\epsilon_N = \frac{1.5^6 - 1}{N} \approx \frac{10}{N} \quad (7)$$

In [30] Bodlund shows the measured normalized covariance δ_p^2 and compares with Lubman's theoretical curve in [1]. Bodlund has normalized on σ_θ before using Lubman's formula. Then he gets the right order of magnitude of C_θ and uses:

$$\delta' = \left(\frac{\sin kr}{kr} \right)^2 \quad (8)$$

We reproduce Bodlund's figure here.

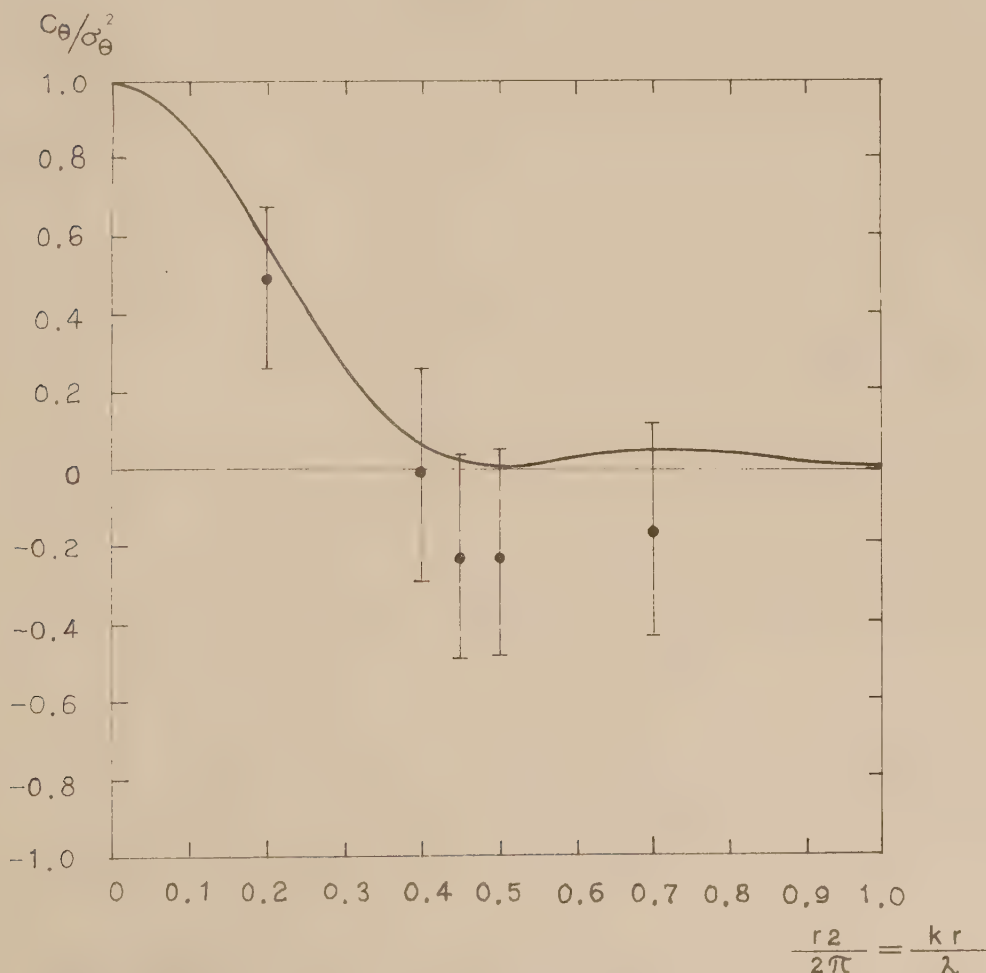


FIGURE 6.7.1. Measured $\delta(r)$ and $\delta'(r)$ according to eq.(8).

Bodlund's comment is that his observations must give a $\delta(r)$ with sign shifts.

Now we shall use Bodlund's measured values again, but compare with the curves according to eqs.(6.6.9) and (6.6.10).

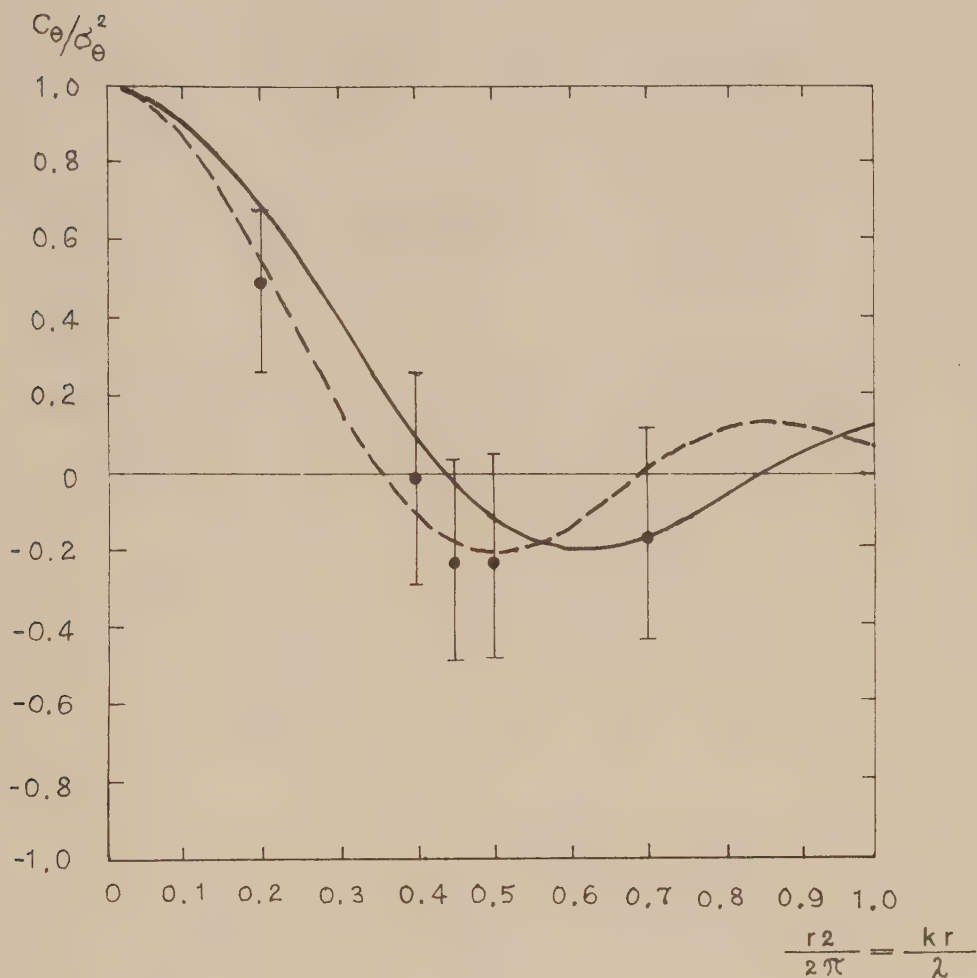


FIGURE 6.7.2. $\delta_\theta(r)$ for 2- and 3-mode according to eqs.(6.6.9) and (6.6.10).

There seems to be a fairly good agreement between measurements and theory in FIG 6.7.2, compared to the case in FIG 6.7.1.

We can combine the approximate results of par. 6.6 with the result of eq.(6). I suggest that a weighted sum of the approximate $\delta_\theta(r)$ for $M = 1, 2$ and 3 , is a useful function when estimating the normalized covariance for the complex field.

The weighting can be performed by using the relative number of modes of

different types in the frequency band (see par. 4.3, eqs.(15), (16) and (17):

$$N_B = \sum N_{Bv} \quad (9)$$

$$\begin{aligned} \delta_{\theta}(r) = & \frac{N_{B1}}{N_B} \cdot \frac{\sin(2kr)}{2kr} + \frac{N_{B2}}{N_B} \cdot \frac{\sin(2kr/\sqrt{2})}{2kr/\sqrt{2}} + \\ & + \frac{N_{B3}}{N_B} \cdot \frac{\sin(2kr/\sqrt{3})}{2kr/\sqrt{3}} \end{aligned} \quad (10)$$

Bodlund's measurements are made in an almost cubic 200 m³ room. In the 125 Hz third octave ($k = 2.8$, $k_B = 0.65$) we get:

N_{Bv}	N'_{Dv}	N_B
13	0.12	
17	0.16	
79	0.72	
$N_B = 109$		

The resulting curve according to eq.(10) is shown in the following figure together with Bodlund's results.

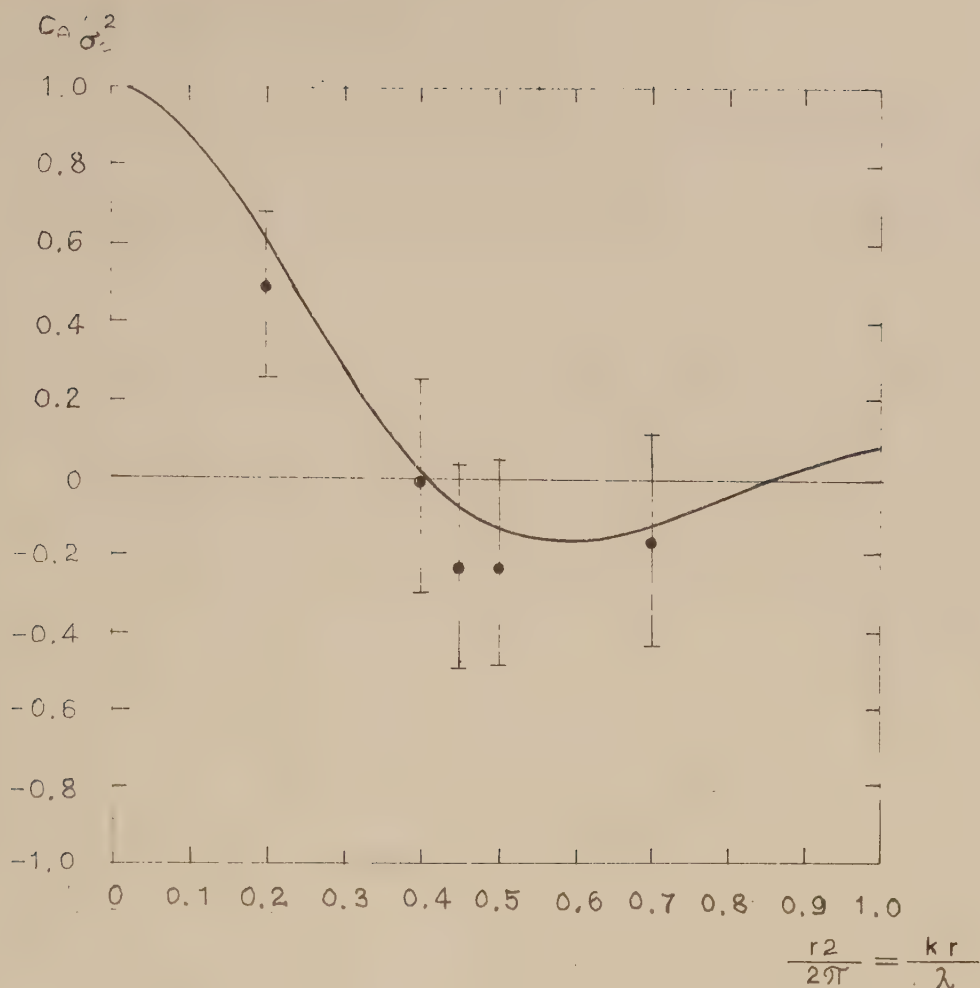


FIGURE 6.7.3. $\delta_{\theta N}(r)$ according to eq.(10).

My approximate formulation can of course be completed by computer evaluation of the complete connections given in par. 6.6 and 6.7. The deviations from $(\sin kr/(kr))^2$ within a broader band have been discussed in literature, but of course this has minor importance compared to the fact that $(\sin kr/kr)^2$ is not the proper covariance function of θ .

6.8. The distributions

The shown result confirms that $C_\theta(r) \neq 2R_q^2(r)$ [1] and also that $\delta_\theta(r) \neq \delta_q^2(r)$, the relation tested by Bodlund. We can get more information concerning these relations from the probability density functions f_q , f_{q^2} and f_D .

The distribution that leads to $C_{q^2} = 2R_q^2$ is very special and different from the usually obtained probability density function, which is rather symmetrical about a peak near μ . If q is normally distributed we have the following probability density for q and q^2 . $f(q)$ is symmetrical, but $f(q^2)$ is very distorted about $\mu_{q^2} = \sigma_q^2$:

$$f_q(q) = \frac{1}{\sqrt{2\pi} \sigma_q} \cdot e^{-q^2/(2\sigma_q^2)} \quad (1)$$

$$f_{q^2}(q^2) = \frac{1}{\sqrt{2\pi} \sigma_q} \cdot \frac{e^{-q^2/(2\sigma_q^2)}}{\sqrt{q^2}} ; q^2 > 0 \quad (2)$$

The probability density of D is quite different from that of q^2 and more like the usual picture of f with a peak near μ_D .

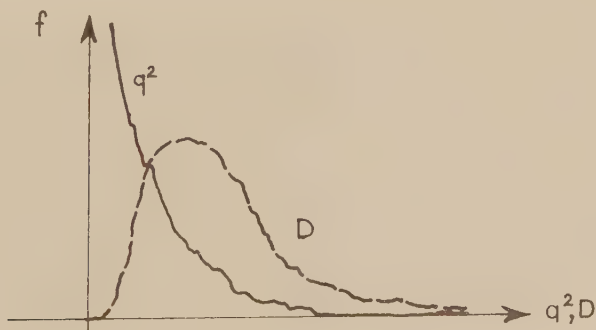


FIGURE 6.8.1. Measured distributions for q^2 and D .

The unsymmetrical $f_{q^2}(q^2)$ -curve explains that $C_{q^2} \geq 0$. D is not the square of a normally distributed variable and $C_D \neq 2R_q^2$. Nevertheless $C_D > 0$, as

C_D is an integral value of R_q^2 .

We can also observe that $f_\theta(\theta)$ in a mode is of sine type:

$$f_\theta(\theta) = \frac{1}{\pi\theta\sqrt{1-\theta}} \quad (3)$$

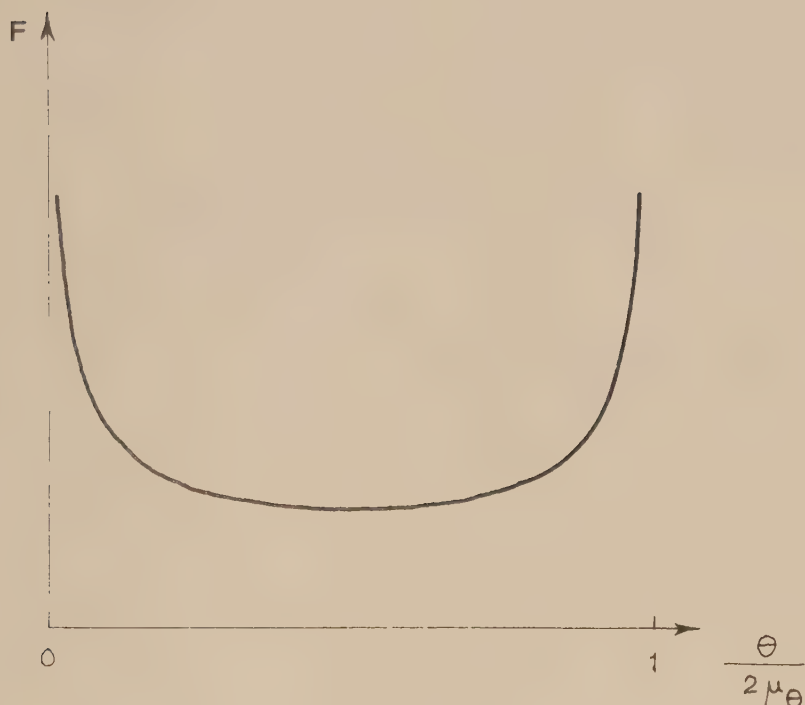


FIGURE 6.8.2. Probability distribution of $\cos^2 x$.

The distributions give another picture of the difference between q^2 and θ . From this we can only conclude that the new covariance function is more natural than $(\sin kr/kr)^2$. The Gaussian character is eliminated in θ . Instead the mode distributions dominate and give a different probability density function. The relative symmetry about μ_θ in the space domain explains the sign shifts of $C_\theta(r)$. It seems as if only the square of a normally distributed signal can give $C \geq 0$ for all values.

In measurements of $C_\theta(r)$ we have to use $C_D(r)$. Then the measuring time must be very long and the microphone movement very slow, to get $D \approx \theta$ and $\sigma_D^2(T) \ll \sigma_\theta^2$. The condition for the first statement is that the microphone speed gives $v_\mu < \lambda/T$, as θ must not be averaged. The second condi-

tion is that $BT \ll 1/\epsilon_N^2$. Below the time distribution, f_D , and the time and space distribution, $f_{D\theta}$, are shown. The condition concerning averaging of θ is fulfilled, but the integration time is only 0.5 s, so the difference between the two probability density curves is small. If $D(r)$ from a measurement of this type is used for the estimation of $C_\theta(r)$, too much of the time variations is affecting the result. Hence the proper $C_\theta(r)$ will not be shown, but a function similar to $C_{q^2}(r)$ is instead the result. This may be the reason why the proper form of $C_\theta(r)$ has not been discovered earlier.

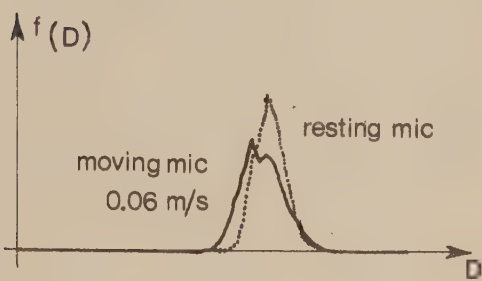


FIGURE 6.8.3. Small difference between f_D and $f_{D\theta}$.

6.9. Continuous and discrete spatial averaging

The new space scanning microphone has led to some comparative studies of the variance of continuous and discrete spatial averaging.

In [57] the deduction of $\text{Var}[\overline{q^2}]$ is made from the following covariance, which has been discussed earlier:

$$\delta'_\theta = \left(\frac{\sin(kr)}{kr} \right)^2 \quad (1)$$

When calculating the variance of the mean value θ_d we shall use the new covariance δ_θ . For M measuring points we get:

$$\beta_d^2 = \frac{\text{Var}[\theta_d]}{\text{Var}[\theta]} = \frac{1}{M} + \frac{2}{M^2} \sum_{m=1}^M \sum_{n=m+1}^M \delta_\theta |\vec{r}_m - \vec{r}_n| \quad (2)$$

Hence the normalized standard error of the discretely formed mean value is:

$$\epsilon_{\theta_d}^2 = \beta_d^2 \cdot \epsilon_N^2 \quad (3)$$

Because of the sign shifts of the covariance of δ_θ the results will be somewhat different from the results of [57], where δ'_θ is used. β_d can be $< 1/M$ for a two-point value, if δ_θ is negative. In [57] zero is the lowest value of δ'_θ . For the approximate result of 3-modes the first minimum of the δ_θ -curve occurs as

$$2kr/\sqrt{3} = 3\pi/2 \rightarrow kr \approx 1.25\pi \approx 4 \quad (4)$$

This can be compared to the first zero of $\sin(kr)$ at $kr = \pi$, used in [57]. The minimum value of δ is $\sin(3\pi/2)/(3\pi/2) = -0.21$.

The optimal measuring distance is not $\approx \lambda/2$, but $\approx 1.2 \cdot \lambda/2$ and we get a minimum for β_d , $M = 2$:

$$\beta_{d,\min}(M = 2) = \frac{1}{2} - \frac{2}{4} \cdot 0.2 = 0.45 \quad (5)$$

We assume that the time variance can be added, as the form of θ in the room mode is independent of time. We refer to the time variance as θ_D :

$$\begin{aligned} \epsilon_d^2(T) &= \epsilon_{Dd}^2 + \epsilon_{\theta d}^2 = \frac{4}{T_1 M} \int_0^{T_1} R_q^2(\tau) d\tau + \beta_d^2 \cdot \epsilon_N^2 = \\ &= \frac{4}{T} \int_0^{T_1} R_q^2(\tau) d\tau + \beta_d^2 \cdot \frac{10}{N} \approx \frac{1}{BT} + \beta_d^2 \cdot \frac{10}{N} \end{aligned} \quad (6)$$

We consider the continuous averaging as a limit case for many close points. When the measuring time is limited we cannot obtain $\overline{q^2} \rightarrow \theta$ in every point. However, it is natural to assume that the close points are so equal that they "help each other" in the time domain, if the spatial scanning is slow, so that $\omega T \gg kL$:

$$\beta_L^2 = \frac{2\epsilon_N^2}{L} \int_0^1 \left(1 - \frac{x}{L}\right) C_\theta(x) dx \quad (7)$$

As the measuring time T_L is finite we must add the time estimate variance and we shall do this independently, as we did to obtain eq.(6):

$$\epsilon_L^2 \approx \epsilon_{DL}^2 + \epsilon_{\theta L}^2 \approx \frac{4}{T_L} \int_0^{T_L} R_q^2(\tau) d\tau + \beta_L^2 \cdot \frac{10}{N} \approx \frac{1}{BT_L} + \beta_L^2 \cdot \frac{10}{N} \quad (8)$$

As an example we choose the reverberant room and $B_F = 850 - 1150$ Hz. The mode bandwidth is $\approx 2.2/T \approx 0.3$, so $B_F/B_{\text{mode}} \approx 1000$. Then B_{T_1} must be much larger than 100 if $\epsilon_{d\theta}^2$ shall dominate over ϵ_{Dd}^2 . $\epsilon_{\theta d} = 3\epsilon_{Dd}$ for $T_1 = 10 \cdot 100/B = 3$ s.

The conditions for continuous averaging are more difficult to give. We have seen how $f_{D\theta}$ was dominated by f_D at a microphone speed of one wave-

length per second. A reasonable demand is that the time within $\lambda/8$ is the same as for one point in discrete averaging, if $\epsilon_{\theta L}^2$ shall dominate over ϵ_{DL}^2 . This is the case for a given L . With the same T_L a longer L is better, as $\epsilon_{\theta L}$ is smaller for increasing L .

6.10. Concluding remarks on Part 6

The special relations between a normally distributed signal and the signal squared have been shown to be valid for the sound pressure in time and space domain.

The true mean square for long measuring time approaches the point expected value θ of the pressure squared, which has been shown to be constant in a field of uncorrelated travelling waves. The space correlation of the variable θ has therefore been studied in modes of different types. The result is covariance functions for θ , which are claimed to be more correct than the functions presented earlier. These functions seem to be valid for the pressure squared (q^2) instead of the point expected value of the pressure squared (θ).

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